# Physics 4311: Thermal Physics - Exam 1

Thursday, March 6, 2025 150 point total

**Problem 1: Short questions** (10 points each = 30 points)

a) You have 5 independent magnetic atoms. The magnetic moment of each atom points "up" with probability 3/4 or "down" with probability 1/4. Find the probability that the moments of 3 of the 5 atoms point "down" while the moments of the other 2 atoms point "up".

b) A box of volume $V$ contains an ideal gas in equilibrium at temperature $T$. The gas is now isothermally compressed to volume $V/2$. Find the ratio $\langle v_f \rangle / \langle v_i \rangle$ between the average speeds of the particles before and after the compression.

c) A container contains a mixture of two ideal gases in equilibrium at temperature $T$, consisting of $N$ atoms of type A of mass $m_A$ and $N$ atoms of type B of mass $m_B$ with $m_A > m_B$. A small hole is made in the wall of the container, and the gas starts escaping. After some time, the number of A atoms remaining in the box is **smaller than / equal to / larger than** the number of B atoms remaining (circle one). Give a short (one or two sentence) explanation.

**Problem 2: Low-speed molecules** (40 points)

Consider a three-dimensional ideal gas of molecules of mass $m$ at temperature $T$. The goal of this problem is to estimate the fraction of molecules whose kinetic energy is lower than $0.05 k_B T$.

a) Find the speed $v_{\max}$ of a molecule of kinetic energy $0.05 k_B T$ in terms of $T$ and $m$.

b) Write down the three-dimensional Maxwell-Boltzmann velocity distribution $P(v_x, v_y, v_z)$

c) Write down an integral for the probability of a molecule to have a speed below $v_{\max}$. Transform to spherical coordinates.

d) Solve the integral for the probability of a molecule to have a speed below $v_{\max}$.
**Hint:** The integral over $v$ cannot be solved in closed form. As the energy is below $0.05 k_B T$, the Boltzmann factor can be approximated, $e^{-mv^2/(2k_BT)} \approx 1$.

**Problem 3: Two coupled Ising spins** (40 points)

Consider two coupled spins that can point either up or down. They are described by spin variables $S_1$ and $S_2$ that each can take the values $+1$ (up) or $-1$ (down). The energy of the system is given by $E = -JS_1S_2$ with $J > 0$. The system is in thermal equilibrium at temperature $T$.

a) Write down all microstates of the system and their energies.

b) Calculate the probabilities of finding the system in each of the microstates.

c) Calculate the average energy as a function of $T$.

d) What is the value of the average energy in the limit $T \to 0$?

e) What is the value of the average energy in the limit $T \to \infty$?

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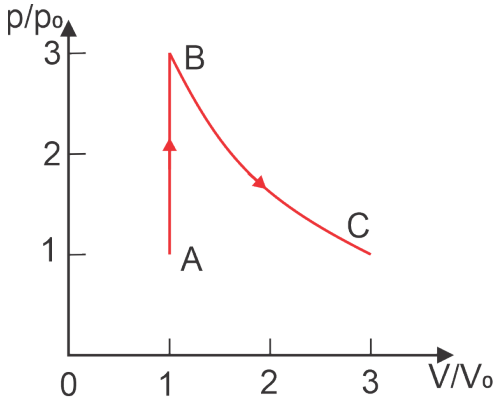
**Problem 4: Gas expansion** (40 points)

An ideal gas of $N$ atoms is taken quasi-statically from point A to B (at constant volume) and then from B to C (at constant temperature) as shown in the pressure-volume diagram. Express all answers in terms of $N$, $k_B$, $p_0$, and $V_0$.




a) Find the temperatures $T_A$, $T_B$ and $T_C$ at points A, B, and C.

b) How much work is done on the gas from A to B?

c) How much heat is flowing into the gas from A to B?

d) How much work is done on the gas during the isothermal expansion from B to C?

e) How much heat is flowing into the gas from B to C?