1a) $$P = \binom{5}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = 10\,\frac{1}{64}\,\frac{9}{16} = \frac{90}{1024} = \frac{45}{512}$$

b) $\langle v\rangle$ depends on T only : $\langle v_f\rangle / \langle v_i\rangle = 1$

c) lighter atoms have higher effusion rate (Graham's law), heavier particles left behind
$\Rightarrow$ more A particles left behind

2a) $\frac{1}{2} m v_{max}^2 = 0.05\, k_B T \qquad v_{max}^2 = 0.1\, k_B T/m$

b) $$p(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{1}{2} m \vec{v}^2 / k_B T}$$

c) $$\underline{P} = \int_{v<v_{max}} dv_x\, dv_y\, dv_z\; p(v_x, v_y, v_z)$$

$$= \int d\varphi \int d\theta \sin\theta \int_0^{v_{max}} dv\, v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{1}{2} m \vec{v}^2 / k_B T}$$

d) $$\underline{P} = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\int_0^{v_{max}} dv\, v^2 = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\frac{1}{3} v_{max}^3$$

$$= \frac{4}{3}\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\left(0.1\,\frac{k_B T}{m}\right)^{3/2}$$

$$= \frac{4}{3}\pi\left(\frac{1}{2\pi}\right)^{3/2}(0.1)^{3/2} = \frac{4}{3}\frac{1}{\sqrt{\pi}}(0.05)^{3/2}$$

3 a) $++ \ , +- \ , -+ \ , --$

$$E_{++} = -J = E_{--}$$

$$E_{+-} = +J = E_{-+}$$

b)
$$\rho_{++} = \frac{1}{Z} e^{+\beta J} = \rho_{--}$$

$$\rho_{+-} = \frac{1}{Z} e^{-\beta J} = \rho_{-+}$$

$$Z = 2e^{+\beta J} + 2e^{-\beta J} = 4\cosh(\beta J)$$

c) $$\langle E \rangle = 2(-J)\frac{1}{Z}e^{\beta J} + 2J\frac{1}{Z}e^{-\beta J}$$

$$= -J\tanh(\beta J)$$

d) $T \to 0, \ \beta \to \infty \qquad \langle E \rangle \to -J$

e) $T \to \infty, \ \beta \to 0 \qquad \langle E \rangle \to 0$

4a) $T_A = \frac{p_0 V_0}{N k_B}$ $\qquad T_B = T_C = \frac{3 p_0 V_0}{N k_B}$

b) $\Delta W = 0 \qquad (\Delta U = 0)$

c) $\Delta Q = \Delta U = \frac{3}{2} N k_B T_B - \frac{3}{2} N k_B T_A$

$$= \frac{3}{2} N k_B \frac{2 p_0 V_0}{N k_B} = 3 p_0 V_0$$

d) $$\Delta W = \int_{V_B}^{V_C} -p\,dV = \int_{V_0}^{3V_0} - \frac{N k_B T_B}{V}\,dV$$

$$= - \frac{3 p V_0}{N k_B} N k_B \int_{V_0}^{3V_0} \frac{dV}{V} = -3 p_0 V_0 \ln 3$$

e) $\Delta U = 0 \Rightarrow \Delta Q = -\Delta W = 3 p_0 V_0 \ln 3$