Physics 4311: Thermal Physics - Exam 2

Thursday, Apr 16, 2025 150 point total + 10 BONUS points

Problem 1: Thermodynamic potentials and Maxwell relations (30 points)

Consider a magnetic material for which the first law reads $dU = T\,dS - m\,dB$. You can treat the magnetization m and the magnetic field (induction) B as scalars.

a) Derive the enthalpy, Helmholtz free energy, and Gibbs free energy as well as their total differentials by performing the appropriate Legendre transformations.

b) Derive all four Maxwell relations for this magnetic material.

Problem 2: Heat capacity at constant tension force (25 points)

The equation of state of an elastic rod can be approximated by $f = \alpha T(L - L_0)$ where f is the tension force, T is the temperature, and L is the length. α and the unstretched length L_0 are constants. The internal energy of the rod is given by $U = C_L T$ where C_L is another constant. Starting from the first law, find the heat capacity C_f at fixed tension force (in terms of L and the constants).

Problem 3: Heat pump (25 points)

A house is heated by an ideal heat pump consisting of a Carnot cycle (running backwards). Over the period of an hour, it removes heat Q_l from the outside at the lower temperature T_l and discharges heat Q_h into the house at the (higher) room temperature T_h, consuming electric energy (work) E. The amount of heat leaking out of the house through walls and windows per hour is $Q_{\text{loss}} = A(T_h - T_l)$ where A is a constant. The heat pump has run for a while, and the inside of the house has reached a steady state.

Derive an expression for the energy E required to run the heat pump (in the steady state) as a function of T_l, T_h, and A.

Hint: You may start from the efficiency of a Carnot cycle running forward (as heat engine), $|W|/Q_h = -W/Q_h = 1 - T_l/T_h$, or some equivalent relation for the Carnot cycle.

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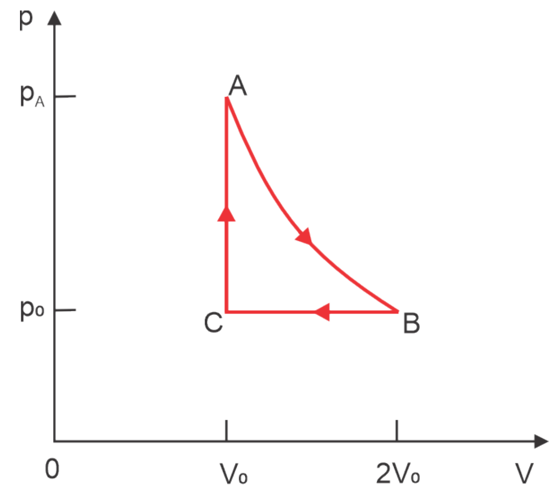
Problem 4: Heat engine (70 points + 10 BONUS points)

A heat engine uses an ideal gas of N atoms as working medium. It undergoes the cycle shown in the figure which consists of an **adiabatic** expansion (A $\to$ B), an **isobaric** compression (B $\to$ C), and an **isochoric** heating (C $\to$ A).



a) Using the equation for an adiabatic curve, find the pressure p_A in terms of p_0.

b) Find the temperatures T_A, T_B, and T_C at points A, B, and C in terms of p_0, V_0, and N.

c) Compute the change in internal energy ΔU, the work ΔW, and the absorbed heat ΔQ for the adiabatic process A $\to$ B (in terms of p_0, V_0).

d) Compute ΔU, ΔW, and ΔQ for the isobaric process B $\to$ C (in terms of p_0, V_0).

e) Compute ΔU, ΔW, and ΔQ for the isochoric process C $\to$ A (in terms of p_0, V_0).

f) (10 BONUS points) The efficiency of the heat engine is defined as $\eta = |W|/Q_{in}$, where Q_{in} is the total "input" heat that is absorbed by the engine (i.e. heat terms that are positive). $|W|$ is the total work done by the engine. Compute the efficiency.