1 a) $dU = T\,dS - m\,dB$

$H = \underline{U} + Bm \qquad dH = dU + d(Bm)$

$\qquad = T\,dS + B\,dm$

$F = \underline{U} - TS \qquad dF = -S\,dT - m\,dB$

$G = \underline{U} - TS + Bm \qquad dG = -S\,dT + B\,dm$

b) $\left(\frac{\partial T}{\partial B}\right)_S = -\left(\frac{\partial m}{\partial S}\right)_B$

$\left(\frac{\partial T}{\partial m}\right)_S = \left(\frac{\partial B}{\partial S}\right)_m$

$\left(\frac{\partial S}{\partial B}\right)_T = \left(\frac{\partial m}{\partial T}\right)_B$

$\left(\frac{\partial S}{\partial m}\right)_T = -\left(\frac{\partial B}{\partial T}\right)_m$

2)

$$d\underline{u} = \delta Q + \delta W = \delta Q + f\,dL$$

$$C_f = \left(\frac{\delta Q}{\partial T}\right)_f = \left(\frac{\partial u}{\partial T}\right)_f - f\left(\frac{\partial L}{\partial T}\right)_f$$

$$\left(\frac{\partial u}{\partial T}\right)_f = C_L \qquad\qquad L = L_0 + \frac{f}{\alpha T}$$

$$\left(\frac{\partial L}{\partial T}\right)_f = \frac{\partial}{\partial T}\left(\frac{f}{\alpha T}\right) = -\frac{f}{\alpha T^2}$$

$$C_f = C_L + \frac{f^2}{\alpha T^2} = C_L + \alpha\left(L - L_0\right)^2$$

3) $|Q_h| = A(T_h - T_c)$

$$1 - \frac{T_c}{T_h} = \frac{-W}{Q_h}$$

$$E = W = -Q_h\left(1 - \frac{T_c}{T_h}\right) = A(T_h - T_c)\left(1 - \frac{T_c}{T_h}\right)$$

$$= A\,\frac{(T_h - T_c)^2}{T_h}$$

4) $\gamma = 5/3$

a) $pV^{\gamma} = \text{const}$

$p_A V_0^{\gamma} = p_0 (2V_0)^{\gamma}$

$p_A = 2^{\gamma} p_0$

b) $T_A = p_A V_A / N k_B = 2^{\gamma} p_0 V_0 / N k_B$

$T_B = 2 p_0 V_0 / N k_B$

$T_C = p_0 V_0 / N k_B$

c) $\delta Q = 0 \Rightarrow \Delta W = \Delta U = \frac{3}{2} N k_B (T_B - T_A)$

$\Delta W = \Delta U = \frac{3}{2}(2 - 2^{\gamma}) p_0 V_0$

d) $\Delta U = \frac{3}{2} N k_B (T_C - T_B) = -\frac{3}{2} p_0 V_0$

$\Delta W = -\int_{2V_0}^{V_0} p\,dV = p_0 V_0$

$\Delta Q = \Delta U - \Delta W = -5/2\, p_0 V_0$

e) $\Delta U = \frac{3}{2} N k_B (T_A - T_C) = \frac{3}{2}(2^{\gamma} - 1) p_0 V_0$

$\Delta W = 0$

$\Delta Q = \Delta U$

BONUS

f) $\eta = \frac{|W|}{Q_{in}} = \frac{\left| \frac{3}{2}(2-2^{\gamma}) p_0 V_0 + p_0 V_0 \right|}{\frac{3}{2}(2^{\gamma} - 1) p_0 V_0}$

$= \frac{|2 - 2^{\gamma} + \frac{2}{3}|}{2^{\gamma} - 1} = \frac{2^{\gamma} - 2 - 2/3}{2^{\gamma} - 1}$

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