Physics 4311: Thermal Physics - Final exam

Friday, May 16, 2025 200 point total

Problem 1: Short questions (10 points each = 30 points)

These questions should not require calculations except, perhaps, one or two lines of math.

a) Consider two boxes A and B of equal volume. Box A contains a monoatomic classical ideal gas, and box B contains a diatomic ideal gas. The two gases have equal particle numbers and are in thermal equilibrium with each other. Which gas has the (i) higher pressure, (ii) higher internal energy? Give a short explanation (three sentences max).

b) A system with three states, with energies E_1, E_2, and E_3 is in equilibrium at temperature T. The energies fulfill $E_1 = E_2 > E_3$. What values does the system's entropy take for $T \to 0$ and $T \to \infty$?

c) An ideal gas is expanded from volume V to volume $2V$. Which process leads to a larger decrease in pressure, an isothermal expansion or an adiabatic expansion? Give a short explanation for your answer.

Problem 2: New state of matter (40 points)

Consider a gas characterized by pressure p, volume V, and temperature T.

a) Show that the internal energy U fulfills the relation $(\partial U/\partial V)_T = -p + T(\partial p/\partial T)_V$. [Hint: Start from the first law and use an appropriate Maxwell relation to replace $(\partial S/\partial V)_T$ by a more useful derivative.]

b) You have discovered a new kind of gas whose thermodynamic and caloric equations of state read

$$pV = AT^3 \quad , \qquad\qquad U = BT^n \ln(V/V_0)$$

where A, B, n and V_0 are constants. Use part a) of the problem to find for what value of n the thermodynamic and caloric equations of state are compatible with each other.

c) Find the constant B in terms of A.

Problem 3: Ultra-relativistic classical ideal gas (60 points)

Consider a gas of N non-interacting, indistinguishable, classical particles at temperature T in a cubic box of linear size L. The particles are ultra-relativistic, i.e., their energy depends on the momentum via $E = \mathrm{c}|\vec{\mathrm{p}}|$, where c is the speed of light.

a) Calculate the partition function and the free energy of the gas. [Hint: Work in spherical coordinates in momentum space.]

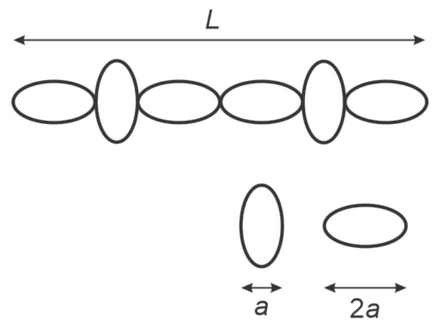
b) Calculate the pressure as function of N, T, and V.

c) Find the internal energy U and the specific heat C_V at constant volume.

d) Determine the specific heat C_p at constant pressure, and compare the adiabatic index $\gamma = C_p/C_V$ to that of the nonrelativistic ideal gas.

Problem 4: One-dimensional polymer (70 points)

A one-dimensional polymer is formed by connecting N ellipsoid-shaped molecules into a one-dimensional chain. Each molecule has two ways of connecting to the polymer (as shown in the figure). It can align either its long axis (length $2a$) or its short axis (length a) with the direction of the polymer chain. A molecule connected along the long axis has energy $E_1 = 0$, a molecule connected along the short axis has energy $E_2 = \epsilon$ with $\epsilon > 0$. The polymer is in equilibrium at temperature T.



a) Find the partition function and the free energy of the polymer.

b) Compute the internal energy as a function of temperature T.

c) Find the values of the energy in the limits $T \to 0$ and $T \to \infty$.

d) Compute the entropy as a function of temperature.

e) What are the values of the entropy in the limits $T \to 0$ and $T \to \infty$?

f) Calculate the average length L of the polymer as a function of T.

g) Find the values of L in the limits $T \to 0$ and $T \to \infty$.

$\int_{-\infty}^{\infty} e^{-\frac{1}{2}(x-x_0)^2/\sigma^2} = (2\pi\sigma^2)^{1/2}$

$\int_0^\infty dx\, x e^{-ax} = 1/a^2$, $\quad \int_0^\infty dx\, x^2 e^{-ax} = 2/a^3$, $\quad \int_0^\infty dx\, x^3 e^{-ax} = 6/a^4$,

$\int_0^\infty dx\, x^4 e^{-ax} = 24/a^5$