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1 a) (i) same pressure $pV = Nk_BT$

(ii) diatomic gas has higher energy (extra d.o.f.)

b) $S = 0$ for $T = 0$ (unique ground state)

$S = k_B \ln 3$ for $T \to \infty$ (3 equally likely states)

c) adiabatic expansion leads to larger p decrease because T decreases

2 a) $dU = TdS - pdV$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

Maxwell for F

$dF = -SdT - pdV$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

b) $$\left(\frac{\partial U}{\partial V}\right)_T = BT^n \frac{1}{V}$$

$$T\left(\frac{\partial p}{\partial T}\right)_V - p = T\frac{\partial}{\partial T}\left(\frac{AT^3}{V}\right)_V - \frac{AT^3}{V}$$

$$= \frac{3AT^3}{V} - \frac{AT^3}{V} = 2\frac{AT^3}{V}$$

$\Rightarrow n = 3$

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c) $B = 2A$

3 a) $Z_N = \frac{1}{N!} Z_1^N$

$$Z_1 = \int \frac{d^3p\, d^3q}{h^3} e^{-\beta c|\vec{p}|} = \frac{V}{h^3} 4\pi \int_0^\infty dp\, p^2 e^{-\beta c p}$$

$$= \frac{V}{h^3} 4\pi \frac{2}{(\beta c)^3} = 8\pi \frac{V}{(\beta c h)^3}$$

$$F = -k_B T \ln Z_N = k_B T \ln(N!) - N k_B T \ln Z_1$$

b) $p = -\left(\frac{\partial F}{\partial V}\right)_T = N k_B T \frac{\partial}{\partial V} \ln Z_1 = N k_B T \frac{\partial}{\partial V} \ln V$

$$p = \frac{N k_B T}{V}$$

c) $U = -\frac{\partial}{\partial \beta} \ln Z_N = -N \frac{\partial}{\partial \beta} \ln Z_1 = -N \frac{\partial}{\partial \beta} \ln \frac{1}{\beta^3}$

$$= +3N \frac{1}{\beta} = 3 N k_B T$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 3 N k_B$$

d) $C_p = \left(\frac{\delta Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p$

$V = \frac{N k_B T}{p}$

$\left(\frac{\partial V}{\partial T}\right)_p = \frac{N k_B}{p}$

$$= 3 N k_B + N k_B$$

$$C_p = 4 N k_B$$

$\gamma = C_p / C_V = 4/3$ (as opposed to $5/3$ for non relativistic case)

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V2

4 a) $Z_N = Z_1^N$

$Z_1 = e^{-\beta E_1} + e^{-\beta E_2} = 1 + e^{-\beta\varepsilon}$

$F = -k_B T \ln Z_N = -N k_B T \ln Z_1 = -N k_B T \ln(1+e^{-\beta\varepsilon})$

b) $U = -\frac{\partial}{\partial\beta} \ln Z_N = -N \frac{\partial}{\partial\beta} \ln(1+e^{-\beta\varepsilon})$

$= \dfrac{N\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} = \dfrac{N\varepsilon}{e^{\beta\varepsilon}+1}$

c) $T \to 0 \ (\beta \to \infty) \Rightarrow U = 0$

$T \to \infty \ (\beta \to 0) \Rightarrow U = \frac{1}{2} N\varepsilon$

d) $F = U - TS \qquad S = (U-F)/T$

$S = \dfrac{N\varepsilon}{T} \dfrac{1}{e^{\beta\varepsilon}+1} + N k_B \ln(1+e^{-\beta\varepsilon})$

$= N k_B \left[\dfrac{\beta\varepsilon}{e^{\beta\varepsilon}+1} + \ln(1+e^{-\beta\varepsilon}) \right]$

e) $T \to 0 \ (\beta \to \infty) \qquad S = 0$

$T \to \infty \ (\beta \to 0) \qquad S = N k_B \ln 2$

f) $L/N = p_1 2a + p_2 a \qquad p_1 = \dfrac{1}{1+e^{-\beta\varepsilon}}$

$p_2 = \dfrac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

$= p_1 2a + (1-p_1) a$

$= a + p_1 a = a + \dfrac{a}{1+e^{-\beta\varepsilon}}$

g) $T \to 0 \ (\beta \to \infty) \quad L = 2aN, \quad T \to \infty \ (\beta \to 0) \quad L = \frac{3}{2} a N$