Physics 4311: Thermal Physics - Homework 1

due date: Tuesday, Feb 4, 2025; please upload your solution as a pdf on Canvas

Problem 1: Joint probabilities (13 points)

The random variables x and y are jointly distributed. x can take values 2, 4, or 6, whereas y can take the values 3 or 5. The joint probabilities are given by $p_{xy}(2,3) = 1/8$, $p_{xy}(2,5) = 1/24$, $p_{xy}(4,3) = 1/4$, $p_{xy}(4,5) = 1/12$, $p_{xy}(6,3) = 3/8$, $p_{xy}(6,5) = 1/8$.

a) Check that p_{xy} is properly normalized.

b) Compute the reduced probabilities $p_x(2)$, $p_x(4)$, and $p_x(6)$.

c) Compute the reduced probabilities $p_y(3)$, and $p_y(5)$

d) Compute the conditional probabilities $p_x(2|y=3)$ and $p_x(2|y=5)$.

e) Determine whether or not x and y are statistically independent.

Problem 2: Gaussian distribution (15 points)

The continuous random variable x has the probability density

$$P(x) = C \exp\left[-\frac{(x-x_0)^2}{2A}\right]$$

for all real x (where x_0, A, and C are constants).

a) Find the value of the constant C (in terms of A and $_0$) such that the probability density is properly normalized.

b) Compute the average $\langle x \rangle$, the median x_M and the most probable value x_P.

c) Compute the second moment $\langle x^2 \rangle$ and the variance σ_x^2.

Problem 3: Probability of a 10% density fluctuation (12 points)

Consider two identical boxes, A and B.

a) 20 particles are distributed over the two identical boxes A and B at random. Calculate the probabilities $P(9)$ and $P(10)$ for finding exactly $N_A = 9$ and $N_A = 10$ particles in box A, respectively. Calculate the ratio $P(9)/P(10)$.

b) Repeat the calculations for 200 particles. Compare the probabilities for $N_A = 90$ and $N_A = 100$.

c) Repeat the calculations for 2000 particles. Compare the probabilities for $N_A = 900$ and $N_A = 1000$.

(Hint: If your calculator cannot handle large factorials, you can either use Stirling's approximation formula $n! \approx \sqrt{2\pi n}\, n^n e^{-n}$ or math software such as Wolfram Alpha.)