Physics 4311: Thermal Physics - Homework 10

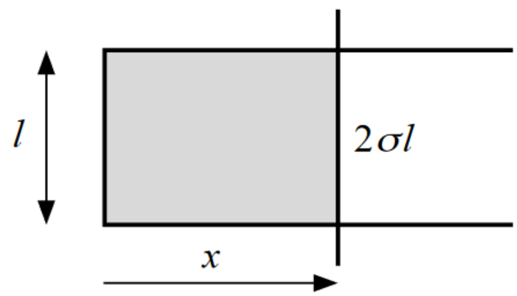
due date: Tuesday, April 15, 2025, please upload your solution as a pdf on Canvas

Problem 1: Maxwell relations for an elastic rod (8 points)

An elastic rod of length L can be stretched or compressed by changing the applied tension force f. The work differential reads $\delta W = f\, dL$. Using the thermodynamic potentials that you found in homework 9.3, derive the four Maxwell relations for this system.

Problem 2: Soap film (16 points)

The figure illustrates a soap film (shown in gray) supported by a wire frame. The right cross wire can slide left or right without friction on the rest of the frame. The soap has a positive surface tension σ; its temperature dependence is given by $\sigma = \sigma_0 - aT$. where σ_0 and a are constants. Note that the film has two surfaces (top and bottom).



a) Derive an expression for the force of the film on the cross wire. What is the direction of this force?

b) Write down the first law, i.e., a relation expressing the change dU in internal energy of the film in terms of the heat TdS absorbed by it and the work done on it when the distance x is changed by an amount dx.

c) Calculate the work done on the film in order to stretch it at a constant temperature T_0 from a length 0 to a length x.

d) Calculate the change in internal energy $\Delta U = U(x) - U(0)$ of the film when it is stretched at a constant temperature T_0 from a length 0 to a length x. [Hint: Use the Maxwell relation arising from the Helmholtz free energy to deal with the heat term in dU).

Problem 3: Thermodynamic and caloric equations of state (16 points)

For a gas or liquid described in terms of pressure p, volume V, and temperature T, show that the thermodynamic equation of state (the relation between p, V, and T) and the caloric equation of state (the dependence of the internal energy U on the other variables) are not independent.

a) Specifically show that $(\partial U/\partial V)_T = -p + T(\partial p/\partial T)_V$

b) Apply the above relation to the ideal gas and show that the internal energy must be volume-independent if $pV = Nk_BT$.

This problems requires a bit of creativity working with partial derivatives. Start from the differential of the entropy as function of U and V. Express the energy differential in terms of T and V. Now use the equality of the mixed second derivatives of the entropy with respect to T and V.