Physics 4311: Thermal Physics - Bonus Homework 13

due date: Friday, May 16, 2025, please upload your solution as a pdf on Canvas

Problem 1: Ideal gas with movable piston (20 points)

A classical ideal gas of N particles at temperature T is in a cylindrical vessel of cross section A. The top of the vessel is closed by a movable piston of mass M.

a) Calculate the partition function for the combined system consisting of gas and piston

b) Determine the average volume of the gas and find the equation of state (pressure-volume-temperature relation)

c) Find the average internal energy and the heat capacity.

d) Discuss which heat capacity you are actually calculating.

Problem 2: Ideal gas in the gravitational field (20 points)

Consider a classical ideal gas of N particles at temperature T in a vessel of cross section A and height H. The particles of the gas are under the influence of a gravitational potential $E_{pot} = mgz$ where m is the mass of a particle, g is the free fall acceleration and z is the vertical coordinate. (Assume H to be large, $mgH \gg k_B T$)

a) Calculate the partition function and the free energy of the gas.

b) Determine the internal energy U and the specific heat C_V. Compare with the equipartition theorem.

c) Calculate how the particle density $n(z)$ changes with z. (Hint: the particle density $n(z)$ is a reduced probability density of the phase space density $p(\vec{r}, \vec{p})$.)

d) What is the approximate ratio between the air density at sea level and at an altitude of 8000m? Use the following approximate values: $g \approx 10\text{ms}^{-2}$, $m \approx 4 * 10^{-26}\text{kg}$ (mass of a nitrogen molecule), $k_B \approx 4/3 * 10^{-23}\text{J/K}$, $T = 300\text{K}$.