Physics 4311: Thermal Physics - Homework 3

due date: Tuesday, Feb 18, 2025, please upload your solution as a pdf on Canvas

Problem 1: Root-mean-square velocity of gas molecules (5 points)

Calculate the root-mean-square velocities of an O_2 molecule room temperature, 293 K, and at a temperature of 1 K.

Problem 2: Space walk (8 points)

An astronaut goes for a space walk, and her space suit is pressurized to 1 atm. Unfortunately, a tiny piece of space dust punctures her suit, and it develops a small hole of radius 10 μm. What force does she feel due to the effusing gas?

Problem 3: High-speed molecules (15 points)

Consider a two-dimensional ideal gas of molecules of mass m at temperature T.

a) Write down the two-dimensional Maxwell-Boltzmann velocity distribution $P(v_x, v_y)$

b) Compute the root-mean-square velocity $\langle \vec{v}^2 \rangle^{1/2}$ in two dimensions.

c) Find the probability of a molecule having a speed larger than twice the root-mean square velocity. (Hint: Go to polar coordinates!)

Problem 4: Doppler broadening of spectral lines (12 points)

The atoms of an interstellar cloud (in thermal equilibrium at temperature T) emit light. The emission frequency of a particular element is ω_0 if the atom is a rest. Due to the thermal motion of the atoms, the observed frequencies are shifted (Doppler effect) to

$$\omega = \omega_0 \left(1 - \frac{v_x}{c}\right)$$

where v_x is the velocity component of the atom away from the detector.

a) Calculate the average observed frequency $\langle \omega \rangle$.

b) Find the width of the spectral line, i.e., the standard deviation σ_ω of the observed frequency.