Physics 4311: Thermal Physics - Homework 4

due date: Tuesday, Feb 25, 2025, please upload your solution as a pdf on Canvas

Problem 1: Mean free paths (10 points)

Consider a gas of nitrogen molecules at room temperature contained in a vacuum chamber. (Take the molecular diameter of N_2 to be 0.37 nm.)

a) Find the collision cross section.

b) Calculate the mean free path and mean collision time at ambient pressure (i.e., before the chamber is evacuated).

c) The chamber is now evacuated to a pressure of 10^{-9} mbar. What are the mean free path and the mean collision time now?

d) The chamber has a diameter of 0.5 m. Estimate how many collisions a molecule will make with the chamber walls compared to collisions with other molecules at a pressure of 10^{-9} mbar.

Problem 2: Pressure change due to effusion (15 points)

A box of volume V contains an ideal gas that is kept at temperature T by a thermostat. Its initial pressure is p_0. At time $t = 0$, a small hole of cross section area A is opened in the box, and gas starts escaping. (Assume that the hole is sufficiently small so that the gas in the box remains in equilibrium during the effusion process.)

a) Derive a differential equation for the pressure p in the box as a function of the time t after opening the hole.

b) Solve the differential equation and find $p(t)$.

Problem 3: Exact differentials (15 points)

a) Test whether the following differentials are exact.

$$\begin{aligned} du_a &= x\,dx + 2\,dy \\ du_b &= -x\,dy \end{aligned}$$

b) If the differential is exact, calculate the indefinite integral.

c) Check the dependence of the integral on the path of integration by explicitly integrating both differentials from point $(x_i, y_i) = (0, 0)$ to $(x_f, y_f) = (2, 2)$ on two different path, $(0, 0) \to (2, 0) \to (2, 2)$ and $(0, 0) \to (0, 2) \to (2, 2)$. Compare the results of the two path and that of a calculation using the indefinite integral (if it exists).