

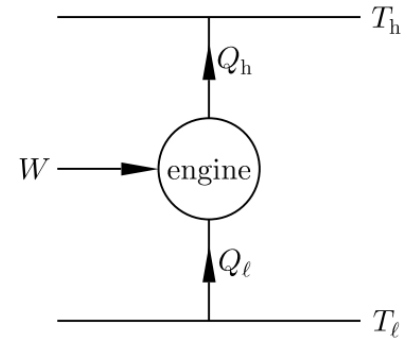
# Physics 4311: Thermal Physics - Homework 7

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due date: Tuesday, March 18, 2025, please upload your solution as a pdf on Canvas

## Problem 1: Heat pump (16 points)

A heat pump is a device that uses work to transport heat from a low-temperature reservoir to a high-temperature reservoir (see figure on the right). Its efficiency can be defined as the ratio of the heat deposited into the high-temperature reservoir and the work done on the system,  $\eta = |Q_h|/W$ .



- Consider a heat pump consisting of a Carnot engine running backwards. Using the results for the ideal-gas based Carnot engine discussed in class, find the efficiency of this “ideal” heat pump in terms of  $T_h$  and  $T_l$ .
- Is the efficiency larger or smaller than unity? Explain what the result means.
- Heat pumps can be used to heat buildings. What is the efficiency of an ideal (Carnot) heat pump that takes heat from the outside air at  $40^\circ\text{F}$  and transports it to the inside of the building which is at  $70^\circ\text{F}$ ?
- What is the efficiency of the heat pump in part c) if the outside air is at a temperature of  $0^\circ\text{F}$ ?

## Problem 2: Carnot process for a rubber band (24 points)

Consider a rubber band having the equation of state  $f = \alpha LT$  where  $L$  is its length,  $f$  is the tension force,  $T$  is temperature, and  $\alpha$  is a constant. The internal energy is given by  $U = C_L T$  where the heat capacity  $C_L$  at fixed length is a constant. The work differential for a rubber band is  $\delta W = f dL$

- Consider an isothermal stretching of the rubber band from  $L_1$  to  $L_2$ . Compute the work  $W_{12}$  done on the system and the heat  $Q_{12}$  absorbed by the system.
- Consider an adiabatic change of the length from  $L_2$  to  $L_3$ . Find the adiabatic  $f - L$  curves by starting from  $\delta Q = 0$  and integrating the resulting differential equation.
- Sketch a Carnot cycle, consisting of two isothermal changes in  $L$  and two adiabatic changes in  $L$  in the  $f - L$  plane.
- Compute the total work during the cycle and the heat absorbed during the four segments of the cycle.
- Explicitly calculate the efficiency.

Hint: The derivation is analogous to that of the Carnot cycle for the ideal gas, but using the equation of state  $f = \alpha LT$  instead of the ideal gas law. This leads to changes in some of the mathematical expressions.