Physics 4311: Thermal Physics - Homework 8

due date: Tuesday, April 1, 2025, please upload your solution as a pdf on Canvas

Problem 1: Entropy of the ideal gas (12 points)

The equation of state of an ideal gas is $pV = Nk_BT$ with p being pressure, V volume, N the number of particles, k_B the Boltzmann constant, and T the temperature. The internal energy is given by $U = (3/2)Nk_BT$.

a) Start from the first law, $dU = T\,dS - p\,dV$, and derive an expression for the entropy of the ideal gas as a function of T and V.

b) Determine the behavior of S for $T \to 0$. What does the result mean?

Problem 2: Air conditioner (18 points)

An ideal air conditioner consists of a Carnot cycle (running backwards). It absorbs heat from the inside of a house at the lower temperature T_l and discharges heat to the outside at the higher temperature T_h, consuming electric energy E. The heat leaking back into the house through the walls and windows is given by $\Delta Q = A(T_h - T_l)$ where A is a constant.

a) The air conditioner runs continuously, and the temperature in the house has reached a steady state. Derive a relation for the inside temperature T_l in terms of T_h, A, and E.

b) The system is designed such that it runs at half of the maximum electrical energy input if the outside temperature is 86 °F and the inside temperature is 70 °F. What is the highest outside temperature for which the system can maintain an inside temperature of 70°F at full electrical input.

Problem 3: Expansion coefficients (10 points)

Consider an isotropic solid, i.e., a solid that behaves the same way in all directions. Upon changing the temperature, its linear size changes as described by the linear expansion coefficient $\alpha_L = (1/L)dL/dT$. Find the volume expansion coefficient $\alpha_V = (1/V)dV/dT$ for this solid. (Hint: Analyze the change in volume of a cubic sample as its length increases from L to $L + dL$.)