Physics 4311: Thermal Physics - Homework 9

due date: Tuesday, April 8, 2025, please upload your solution as a pdf on Canvas

Problem 1: Additivity of the entropy (10 points)

A system consists of two subsystems A and B. Subsystem A can be in states $i = 1 \dots n$ and subsystem B can be states $j = 1 \dots m$. The Gibbs entropy of this system reads

$$S = -k_B \sum_{i=1}^{n} \sum_{j=1}^{m} p(i,j) \ln p(i,j)$$

where $p(i,j)$ are the joint probabilities for the states of the subsystems.

Show that if the two subsystems A and B are statistically independent, then the entropy S of the total system is the sum of the entropies of the two subsystems.

Problem 2: Maxima of entropy (15 points)

As system can be in N different states with probabilities p_i ($i = 1 \dots N$). Determine which p_i lead to the maximum (Gibbs) entropy under the following constraints:

a) Fixed normalization $\sum_i p_i = 1$

b) fixed normalization $\sum_i p_i = 1$ and fixed average energy $\langle E \rangle = \sum_i p_i E_i$.

Hint: Use Lagrange multipliers to enforce the constraints.

Problem 3: Thermodynamic potentials of elastic rod (15 points)

An elastic rod of length L can be stretched or compressed by changing the applied tension force f. The work differential reads $\delta W = f\, dL$. Start from the first law $dU = T\, dS + f\, dL$ and derive the formulas for the thermodynamic potentials and their total differentials in terms of the natural variables.

a) enthalpy,

b) Helmholtz free energy,

c) Gibbs free energy.