10.1. $dU = TdS + f\,dL$

$F = U - TS \qquad dF = -S\,dT + f\,dL$

$H = U - fL \qquad dH = T\,dS - L\,df$

$G = U - TS - fL \qquad dG = -S\,dT - L\,df$

<u>Maxwell relations</u>

$$\left(\frac{\partial T}{\partial L}\right)_S = \left(\frac{\partial f}{\partial S}\right)_L$$

$$-\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial f}{\partial T}\right)_L$$

$$\left(\frac{\partial T}{\partial f}\right)_S = -\left(\frac{\partial L}{\partial S}\right)_f$$

$$\left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial L}{\partial T}\right)_f$$

10.2) a) move wire to the right by dx

$\Rightarrow dA = 2\ell\, dx$

$\sigma\, dA = f\, dx \qquad f = 2\ell\sigma$ (to left)

b) $dU = T\,dS + 2\ell\sigma\, dX$

c) $W = \int_0^X f\, dx' = 2\ell\sigma \int_0^X dx' = 2\ell\sigma X$

d) $$\Delta U = \int_{x'=0}^{X} dU = \int_{x'=0}^{X} \left(\frac{\partial U}{\partial x}\right)_{T=T_0} dX$$

$$\left(\frac{\partial U}{\partial x}\right)_T = T\left(\frac{\partial S}{\partial x}\right)_T + f = T\left(\frac{\partial S}{\partial x}\right)_T + 2\sigma\ell$$

Maxwell relation for F : $dF = -S\,dT + 2\gamma\ell\, dx$

$$\left(\frac{\partial S}{\partial x}\right)_T = -\left(\frac{\partial}{\partial T}(2\sigma\ell)\right)_x = -2\ell(-a) = 2\ell a$$

$$\left(\frac{\partial U}{\partial x}\right)_T = 2\ell a T + 2\sigma\ell = 2\sigma_0\ell$$

$$U(x) - U(0) = 2\sigma_0\ell X$$

10.3 a) $dU = T\,dS - p\,dV$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$

Maxwell relation for $dF = -S\,dT - p\,dV$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

b) $pV = Nk_BT$ $\qquad p = \frac{Nk_BT}{V}$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{Nk_B}{V} = \frac{p}{T}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\frac{p}{T} - p \doteq 0$$