8.1. a) $dU = T\,dS - p\,dV$

$pV = Nk_BT$

$U = \frac{3}{2}Nk_BT$

$$dS = \frac{dU}{T} + \frac{p}{T}dV$$

$$dS = \frac{3}{2}Nk_B\frac{dT}{T} + Nk_B\frac{dV}{V}$$

$$S = \frac{3}{2}Nk_B\ln\left(\frac{T}{T_0}\right) + Nk_B\ln\left(\frac{V}{V_0}\right) + S_0$$

b) for $T \to 0$, $S \to -\infty$

contradicts condition $S \geq 0$ (follows from $S = k_B \ln \Omega$)

$\Rightarrow$ ideal gas law cannot hold down to $T = 0$

8.2 a) in steady state, heat leaking into house

$\Delta Q = A(T_h - T_e)$ equals heat removed by

ac, Q_e

- relate Q_e to E:

Carnot backwards: $Q_e > 0$, $Q_h < 0$

1st law: $E + Q_h + Q_e = 0$

from class $\left|\frac{Q_h}{Q_e}\right| = \frac{T_h}{T_e} \Rightarrow Q_h = -\frac{T_h}{T_e} Q_e$

$$E + Q_e - Q_e \frac{T_h}{T_e} = E + Q_e \frac{T_e - T_h}{T_e} = 0$$

$$Q_e = \frac{T_e}{T_h - T_e} E$$

- in steady state $Q_e = \Delta Q$

$$\frac{T_e}{T_h - T_e} E = A(T_h - T_e)$$

$$T_e E = A(T_h - T_e)^2 = A T_h^2 - 2A T_h T_e + A T_e^2$$

$$T_e^2 - \left(2 T_h + \frac{E}{A}\right) T_e + T_h^2 = 0$$

$$T_e = T_h + \frac{E}{2A} \pm \sqrt{\left(T_h + \frac{E}{2A}\right)^2 - T_h^2}$$

pick (−) sign because $T_e < T_h$

8.2 b)

$T_e = 70F = 294.3\ K$

$T_h = 86F = 303.2\ K$

$T_h' = ?$

$$\left.\begin{aligned} T_e \frac{E_{max}}{2} &= A(T_h - T_e)^2 \\ T_e E_{max} &= A(T_h' - T_e)^2 \end{aligned}\right\} \quad 2(T_h - T_e)^2 = (T_h' - T_e)^2$$

$$T_h' - T_e = \sqrt{2}(T_h - T_e) = 22.6\ F$$

$$T_h' = 92.6\ F$$

8.3 $V(L) = L^3$

$V(L+dL) = (L+dL)^3 = L^3 + 3L^2 dL + O(dL^2)$

$dV = V(L+dL) - V(L) = 3L^2 dL$

$\alpha_V = \frac{1}{V}\left(\frac{dV}{dT}\right) = \frac{1}{L^3} 3L^2 \left(\frac{dL}{dT}\right) = 3 \frac{1}{L}\left(\frac{dL}{dT}\right) = 3\alpha_L$

$\alpha_V = 3\alpha_L$