Physics 5413: Chaos, fractals, and nonlinear dynamics – Project 2

due date Thursday, Sep 12, 2024

Model of a fishery (100 points)

The equation

$$\dot{N} = rN\left(1 - \frac{N}{K}\right) - H\frac{N}{A+N}$$

provides a simple model of a fishery. N is the number of fish, and r, K, H, A are positive parameters. In the absence of fishing ($H = 0$) the population grows with an initial rate r to a maximum population of $N_{max} = K$. The effect of fishing is modelled by the second term. If only few fish are there, they are taken at a rate HN/A proportional to the number of fish. For large populations, the fishermen cannot keep up, they can fish at a maximum rate H independent of the population.

1. Give a biological interpretation of the parameter A. What does it measure?
2. Show that the system can be rewritten in dimensionless form as

   $$dx/d\tau = x(1-x) - h\frac{x}{(a+x)}$$

   for suitably defined dimensionless quantities x, τ, a, h.
3. Find all fixed points of the system. Show that it can have one, two, or three fixed points, depending on the values of a and h.
4. Determine the stability of the fixed points. Explain their meaning in terms of the fishery problem.
5. Draw the stability diagram of the system in the h-a parameter space. This means, show which fixed point the system approaches for each point in the h-a plane.