

# Physics 6311: Statistical Mechanics - Homework 10

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due date: Tuesday, October 31, 2023

## Problem 1: Generalized Bose gas (20 points)

Consider a gas of noninteracting identical bosons of spin  $S$  in  $d$  dimensions. The single-particle energy-momentum relation is given by  $\epsilon(\mathbf{p}) = A|\mathbf{p}|^z$  with positive prefactor  $A$  and exponent  $z$ .

- Compute the density of states  $g(\epsilon)$ .
- Calculate the maximum possible particle number in excited single-particle states as a function of temperature. For which values of  $d$  and  $z$  does the system show Bose-Einstein condensation?
- If there is Bose-Einstein condensation, evaluate the critical temperature  $T_c$ .
- Find the specific heat for temperatures  $T \leq T_c$ .
- Find the pressure for temperatures  $T \leq T_c$ .

## Problem 2: Background radiation (8 points)

In space there exists a background electromagnetic radiation corresponding to an equilibrium temperature of about 3K. At what frequency is the maximum of the energy density of this radiation? What is the corresponding wavelength?

## Problem 3: Radiation of Betelgeuse (12 points)

The luminosity (total amount of energy emitted per time) of the star Betelgeuse is about  $10^4$  times that of the sun. (The solar luminosity is approximately  $3.828 \times 10^{26}$  W.) The energy density  $u(\epsilon)$  of Betelgeuse's radiation has its maximum at a photon energy  $\epsilon \approx 0.8$  eV.

- Find the surface temperature of Betelgeuse, assuming it emits blackbody radiation.
- Estimate the radius of Betelgeuse.
- Why is Betelgeuse called a red giant?