Physics 6311: Statistical Mechanics - Homework 11

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due date: Tuesday, Nov 7, 2023

Problem 1: Thermodynamics of Magnons (15 points)

Spin waves or magnons are elementary excitations of Bose type in ferromagnetic materials. Their dispersion relation is $\omega = D\ k^2$ for small frequencies $\omega \ll \omega_{max}$. Calculate the contribution of the magnons to the specific heat at low temperatures $k_B T \ll \hbar\omega_{max}$.

(Hints: There is no conservation law for the magnon number, the rest mass is zero. You do not have to evaluate dimensionless integrals if you have shown that they converge.)

Problem 2: Phonons in liquid ^{4}Helium (10 points)

The longitudinal phonons in ^{4}He at low temperatures have a velocity of $c = 238.3$ m/s. Transversal phonons do not exist in liquids. The density is 0.145 g/cm^3.

a) Calculate the Debye temperature of the phonons within the Debye model.

b) Calculate the heat capacity contribution of the phonons and compare to the experimental value of $c_V = 0.0204$ (T/K)^3 J/gK.

Problem 3: Phonons in a 1D chain (15 points)

Consider a one-dimensional chain of atoms (model for a linear molecule). The vibrational part of the Hamiltonian is

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \frac{A}{2} \sum_{i=1}^{N} (x_i - x_{i+1})^2 \ .$$

where x_i is the displacement of atom i from its rest position and m is the mass of one of the atoms. Assume periodic boundary conditions.

a) Explicitly determine the normal modes by diagonalizing H (Hint: Use the Fourier transformation).

b) Calculate energy and specific heat as functions of temperature for low temperatures.