Physics 6311: Statistical Mechanics - Homework 12

due date: Tuesday, November 14, 2023

Problem 1: Liquid ^{3}Helium (8 points)

Liquid ^{3}He is approximately a Fermi gas (spin 1/2). The density is 0.081 g/cm^3.

a) Calculate the Fermi energy (at zero temperature). Also calculate the Fermi velocity (the velocity corresponding to the Fermi energy).

b) At roughly what temperatures do you expect the fermionic character of ^{3}He to be important?

Problem 2: Velocity distribution of the Fermi gas (10 points)

For an ideal Fermi gas at zero temperature, derive the probability density of the particle velocities and compare it to the Maxwell distribution of a classical ideal gas of the same total energy (per particle).

[Hint: You will need to find the correct temperature for the classical gas.]

Problem 3: Fermions on a surface (10 points)

Consider an ideal gas of N spin-1/2 fermions of mass m on a planar surface of area A. Derive a closed form expression for the chemical potential as a function of temperature T (valid for all temperatures). Discuss the limits $T \to 0$ and $T \to \infty$.

Problem 4: Particle number fluctuations of the ideal Fermi gas (12 points)

Consider the variance $\langle N^2 \rangle - \langle N \rangle^2$ of the particle number of an ideal Fermi gas close to zero temperature as a function of temperature and particle number.

a) Derive the general grand canonical relation

$$\langle N^2 \rangle - \langle N \rangle^2 = k_B T \left(\frac{\partial \langle N \rangle}{\partial \mu} \right)_{T,V} \ .$$

analogously to the calculation of the energy fluctuations in the canonical ensemble.

b) Apply this relation to the ideal Fermi gas. Why does the result differ from what you would expect from the central limit theorem?

[Hint: The derivative on the r.h.s. can be evaluated at zero temperature. Why?]