Physics 6311: Statistical Mechanics - Homework 2

due date: Tuesday, Sep 9, 2025

Problem 1: Carnot process for a paramagnetic substance (16 points)

Consider a paramagnetic substance whose equation of state is $M = DH/T$ where T is the temperature, M is the magnetization, H is magnetic field, and D is a material specific constant. The internal energy is $U = CT$ where the specific heat C is a constant.

a) Sketch a typical Carnot cycle in the M-H plane.

b) Compute the total absorbed heat and the total work during one cycle.

c) Calculate the efficiency.

Problem 2: Entropy of the ideal gas (8 points)

The equation of state of an ideal gas is $pV = Nk_BT$ with p being pressure, V volume, N the number of particles, k_B the Boltzmann constant, and T the temperature. The internal energy is $U = (3/2)Nk_BT$. Calculate the entropy of the ideal gas as a function of T and V. What happens for $T \to 0$?

Problem 3: Thermodynamic potentials of elastic rod (16 points)

An elastic rod of length L can be stretched or compressed by changing the applied tension force f. The work differential reads $\delta W = f\,dL$. Start from the first law $dU = T\,dS + f\,dL$ and derive the formulas for the thermodynamic potentials and their total differentials in terms of the natural variables.

a) enthalpy,

b) Helmholtz free energy,

c) Gibbs free energy.

d) Also derive the four Maxwell relations for this system.