Physics 6311: Statistical Mechanics - Homework 2

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due date: Tuesday, Sep 5, 2023

Problem 1: Equivalence of the Carnot and ideal gas temperature scales (20 points)

In class, we have defined the temperature T via the efficiency of a Carnot process. Alternatively, an "ideal gas temperature" Θ can be defined via the ideal gas equation of state, $\Theta = pV/(Nk_B)$, and the caloric equation of state, $U = (3/2)Nk_B\Theta$. Here, p is the pressure, V volume, N the number of particles, U the internal energy, and k_B the Boltzmann constant.

Show that Θ is equivalent to the Carnot temperature T defined via the efficiency of a Carnot process. To do so, explicitly calculate the efficiency of a Carnot process using the above ideal gas as working substance and show that it is $\eta = 1 - \Theta_2/\Theta_1$.

Problem 2: Maxwell relations (10 points)

A dielectric material is characterized by polarization P, electric field E, and temperature T (PET system). Its work differential reads $\delta W = E\, dP$.

Derive the Maxwell relations between various derivatives of thermodynamic quantities. The Maxwell relations follow from the fact that the differentials of the thermodynamic potentials U, H, A and G are exact, if they are expressed in terms of their natural variables.

Problem 3: Response functions of the ideal gas (10 points)

An ideal gas obeys the equation of state $pV = Nk_BT$ with p being pressure, V volume, N the number of particles, k_B the Boltzmann constant, and T the temperature. The internal energy is $U = (3/2)Nk_BT$. Calculate the specific heat at constant volume C_V, the specific heat at constant pressure C_p, the isothermal and adiabatic compressibilities κ_T and κ_S as well as the thermal expansion coefficient α of this ideal gas.