Physics 6311: Statistical Mechanics - Homework 3

due date: September 12, 2023

Problem 1: Box distributions (16 points)

The random variables X and Y are independent and have identical box distributions

$$P_X(x) = \begin{cases} 1 & (0 < x < 1) \\ 0 & \text{otherwise} \end{cases} \quad , \qquad P_Y(y) = \begin{cases} 1 & (0 < y < 1) \\ 0 & \text{otherwise} \end{cases}$$

a) Find the averages $\langle x \rangle$ and $\langle y \rangle$.

b) Compute the variances σ_x^2 and σ_y^2.

c) A new random variable Z is defined as $Z = X + Y$. Find its average $\langle z \rangle$ and variance σ_z^2

d) Derive the probability density $P_Z(z)$ of the random variable Z. (Hint: Use the method of characteristic functions)

Problem 2: Diode (12 points)

The current I across a diode is related to the applied voltage V via

$$I = I_0[e^{eV/(k_B T)} - 1] \ .$$

The diode is subject to a random voltage V which is Gaussian distributed with zero mean and variance σ^2. Calculate the probability density $P(I)$ of the current. Find the most probable current and the average current.

Problem 3: Probability of a density fluctuation (12 points)

Consider two identical boxes, A and B.

a) 10 particles are distributed over the two boxes at random. Calculate the probabilities $P(4)$ and $P(5)$ for finding exactly $N_A = 4$ and $N_A = 5$ particles in the box A, respectively. Calculate $P(4)/P(5)$.

b) Repeat the calculations for 1000 particles. Compare $N_A = 400$ and $N_A = 500$. (Hint: It may be convenient to first compute $\ln[P(400)/P(500)]$ and then re-exponentiate the result. (For large n the factorial can be approximated by Stirling's formula $\ln(n!) \approx n \ln(n) - n$)