Physics 6311: Statistical Mechanics - Homework 5

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due date: Tuesday, Sep 26, 2023

Problem 1: Microcanonical ideal gas (20 points)

Consider a gas of nonrelativistic, non-interacting, distinguishable quantum particles in a cubic box of linear size L (with periodic boundary conditions). The energy-momentum relation of a single particle is $\epsilon = \vec{p}^{\,2}/(2m)$.

a) Determine the (single-particle) wave functions. What are the allowed $\vec{k}$ values? What is the volume per state in $\vec{k}$-space?
b) Calculate the number of microstates as a function of the total (system) energy E. (Hint: First calculate the number of states with energies less than E and then take the derivative with respect to E.)
c) Calculate the entropy as function of the energy.
d) Calculate the temperature and the caloric equation of state (energy-temperature relation).
e) Calculate the thermodynamic equation of state (relation between p, V, T).

Problem 2: Pendulum (20 points)

Consider a (classical) pendulum consisting of a point mass m attached to the pivot point via a massless rod of length L. The pendulum is coupled to a heat bath at temperature T, and its motion is restricted to a vertical plane.

a) Using the canonical ensemble, write down the partition function. Show that it factorizes into kinetic and potential parts.
b) Find the average angular velocity of the pendulum and its standard deviation as functions of temperature.
c) Evaluate the partition function in the limit of small vibrations. Find the average energy and the specific heat of the pendulum in this limit.
d) Calculate the lowest order in T corrections to the small-vibration results for the average energy and the specific heat. (To do so, expand the potential energy about its minimum).