Physics 6311: Statistical Mechanics - Homework 8

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due date: Tuesday, Oct 17, 2023

Problem 1: Atoms on a lattice (40 points)

This is the same problem as problem 4 of homework 4, but this time, you are going to solve it using the grand-canonical ensemble and chemical equilibrium between the regular sites and the interstitial sites.

Consider a lattice having N regular lattice sites as well as N interstitial lattice sites. Each site can be occupied by either 0 or 1 atoms. An atom on a regular site has energy 0 while an atom on an interstitial site has energy ϵ. The whole lattice is now occupied by N atoms and coupled to a heat bath at temperature T. Use the grand-canonical ensemble to analyze this system.

a) Find the number N_r of atoms on the regular sites as a function of the temperature and the chemical potential.

b) Find the number N_i of atoms on the interstitial sites as a function of the temperature and the chemical potential.

c) Find the value of μ for which the total particle number $N_r + N_i$ equals N.

d) Compute the average energy and express it as a function of T and N. Compare with the results of HW 4.4