Physics 6311: Statistical Mechanics - Homework 9

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due date: Tuesday, October 24, 2023

Problem 1: Identical particles in two-level system (15 points)

A quantum mechanical system has two single-particle states $|a\rangle$ and $|b\rangle$ with energies $\epsilon_a = -\epsilon_b = \epsilon$.

a) The system is occupied by two identical particles. Write down all possible states, the corresponding energies and the **canonical** probabilities for these states for bosons (S=0) and for fermions (S=1/2, but both particles being in the $\uparrow$ state). Using the canonical ensemble calculate the Helmholtz free energy, the entropy, the internal energy and the specific heat as functions of temperature.

b) Consider an additional term in the Hamiltonian, viz, an interaction between the particles of the form Un_an_b. where U is the interaction energy and n_a and n_b are the particle numbers of the two single-particle states. How do the canonical probabilities for the two-boson states from a) change as a result of U? Discuss the limits $U \to \infty$ and $U \to -\infty$.

Problem 2: Quantum corrections to classical ideal gas (25 points)

Calculate the lowest order quantum corrections to the internal energy of the classical ideal gas (as functions of particle number and temperature) both for fermions and for bosons.

a) Start from the Bose and Fermi occupation numbers. In the classical limit the average occupation numbers are small compared to unity. Expand the occupation numbers about this limit. Don't forget the corrections to μ.)

b) Express the particle number N and the internal energy E in terms of the (expanded) occupation numbers.

c) Compute the lowest order corrections to the classical result for the energy per particle, E/N.

[Hint: Think about what μ value to use!]