1a) $$Q = e^{+\beta g \mu_B B \frac{1}{2}} + e^{-\beta g \mu_B B \frac{1}{2}}$$

$$= 2 \cosh(\beta B \mu_B g \tfrac{1}{2})$$

$$A = -k_B T \ln\left[2 \cosh(\tfrac{1}{2} \beta B \mu_B g)\right]$$

b) $$\langle S_z \rangle = \frac{1}{\beta g \mu_B} \frac{\partial}{\partial B} \ln Q = \frac{1}{2} \tanh(\tfrac{1}{2} \beta B \mu_B g)$$

$$m = \tfrac{1}{2} g \mu_B \tanh(\tfrac{1}{2} \beta B g \mu_B)$$

c) $T \to 0,\ \beta \to \infty$: $\quad m = \frac{1}{2} g \mu_B$

$T \to \infty,\ \beta \to 0$: $\quad m = \emptyset$

d) $$\chi = \left(\frac{\partial m}{\partial B}\right)_T = \frac{1}{2} g \mu_B \frac{1/2\, \beta g \mu_B}{\cosh^2(\frac{1}{2} \beta B g \mu_B)}$$

$$\chi = \frac{1}{4} \beta g^2 \mu_B^2 \frac{1}{\cosh^2(\frac{1}{2} \beta B g \mu_B)}$$

e) $T \to \infty,\ \beta \to 0$ $\quad$ expand $\cosh^2(\frac{1}{2} \beta B g \mu_B) \approx 1$

$$\chi = \frac{g^2 \mu_B^2}{4 k_B T}$$ Curie law

2 a) $Q_N = Q_1^N = \left(e^{-\beta\varepsilon_0} + e^{-\beta\varepsilon_1}\right)^N$

$= \left(e^{-\beta\varepsilon_0}\right)^N \left(1 + e^{-\beta\varepsilon}\right)^N \qquad \varepsilon = \varepsilon_1 - \varepsilon_0$

b) $p_e = \frac{1}{Q_1} e^{-\beta\varepsilon_0} = \frac{1}{1+e^{-\beta\varepsilon}}$

$p_s = \frac{1}{Q_1} e^{-\beta\varepsilon_1} = \frac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

c) $\langle L \rangle = N\left(p_e 2a + p_s a\right)$

$= N\left(\frac{2a}{1+e^{-\beta\varepsilon}} + \frac{a e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}\right) = Na \frac{2+e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

d) $T \to 0 \quad : \quad \langle L \rangle \to 2Na$

$T \to \infty \quad : \quad \langle L \rangle \to \frac{3}{2} Na$

e) $\frac{5}{4} a \cancel{N} = \cancel{N}a \frac{2+e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

$5\left(1+e^{-\beta\varepsilon}\right) = 8 + 4e^{-\beta\varepsilon}$

$e^{-\beta\varepsilon} = 3 \qquad \varepsilon/k_BT = \ln 1/3 = -\ln 3$

$k_BT = -\varepsilon/\ln 3 < 0$

3a) $$Q_N = \frac{1}{N!} Q_1^N$$

$$Q_1 = V\int \frac{d^3p}{h^3} e^{-\beta A|\vec{p}_n|^s} = \frac{4\pi V}{h^3}\int_0^\infty dp\, p^2 e^{-\beta A|p|^s}$$

$$Q_1 = \frac{4\pi V}{h^3}\frac{1}{s}(\beta A)^{-3/s}\,\Gamma(3/s)$$

b) $$A = -k_BT \ln Q_N = -k_BTN\ln Q_1 + k_BT\ln N!$$

$$p = -\left(\frac{\partial A}{\partial V}\right) = k_BTN/V \qquad pV = Nk_BT$$

c) $$E = -\frac{\partial}{\partial\beta}\ln Q = -\frac{\partial}{\partial\beta}\ln \beta^{-3N/s}$$

$$= \frac{3N}{s}k_BT$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3N}{s}k_B$$

d) $$C_p = \left(\frac{\delta Q}{\partial T}\right)_p = \left(\frac{\partial E + p\partial V}{\partial T}\right)_p = \frac{3N}{s}k_B + Nk_B$$

$$= \left(\frac{3}{s}+1\right)Nk_B$$

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