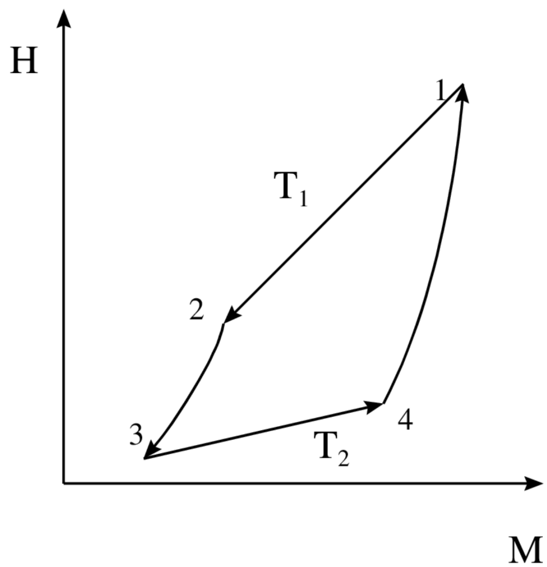
Physics 6311: Statistical Mechanics - HW Solution 2

due date: Tuesday, Sep 9, 2025

Problem 1: Carnot process for a paramagnetic substance (16 points)

a)
- isotherms are straight lines through the origin, $M = DH/T$.
- to find adiabatic $H - M$ curves, start from $0 = \delta Q = CdT - HdM$
  use equation of state to substitute for H giving $CdT = HdM = (MT/D)dM$
  integrate ODE: $CD \ln(T/T_0) = (M^2 - M_0^2)/2$ or equivalently $T/T_0 = \exp[(M^2 - M_0^2)/(2CD)]$
  use equation of state to substitute for T giving $H/H_0 = (M/M_0) \exp[(M^2 - M_0^2)/(2CD)]$



b) **1–2:** isothermal at T_1: $dU = 0$

$$\Delta Q_{12} = -\Delta W_{12} = -\int_{M_1}^{M_2} H dM = -\int_{M_1}^{M_2} (T_1 M/D) dM = -(M_2^2 - M_1^2) T_1/(2D)$$

2–3: adiabatic, $\Delta Q_{23} = 0$
3–4: isothermal at T_2: $dU = 0$

$$\Delta Q_{34} = -\Delta W_{34} = -\int_{M_3}^{M_4} H dM = -\int_{M_3}^{M_4} (T_2 M/D) dM = -(M_4^2 - M_3^2) T_2/(2D)$$

4–1: adiabatic, $\Delta Q_{41} = 0$

c) from the equations of the two adiabatic curves: $(M_2^2 - M_3^2) = (M_1^2 - M_4^2)$

$$\eta = 1 + \frac{\Delta Q_{34}}{\Delta Q_{12}} = 1 + \frac{(M_3^2 - M_4^2) T_2}{(M_1^2 - M_2^2) T_1} = 1 - \frac{T_2}{T_1}$$

Problem 2: Entropy of the ideal gas (8 points)

$pV = Nk_BT$, $U = (3/2)Nk_BT$

$dU = TdS - pdV$

$dS = dU/T + pdV/T = (3/2)Nk_BdT/T + Nk_BdV/V$

$S = (3/2)Nk_B \log(T/T_0) + Nk_B \log(V/V_0) + S_0$

S diverges for $T \to 0$: violation of the third law. Ideal gas ill defined for $T \to 0$.

Problem 3: Thermodynamic potentials of elastic rod (16 points)

a) $dU = TdS + fdL$

b) enthalpy $H = U - fL$, $\quad dH = TdS - Ldf$

c) Helmholtz free energy $A = U - TS$, $\quad dA = -SdT + fdL$

d) Gibbs free energy $G = U - TS - fL$, $\quad dG = -SdT - Ldf$

e) $dU = TdS + fdL$: $\quad (\partial T/\partial L)_S = (\partial f/\partial S)_L$

$dH = TdS - Ldf$: $\quad (\partial T/\partial f)_S = -(\partial L/\partial S)_f$

$dA = -SdT + fdL$: $\quad (\partial S/\partial L)_T = -(\partial f/\partial T)_L$

$dG = -SdT - Ldf$: $\quad (\partial S/\partial f)_T = (\partial L/\partial T)_f$