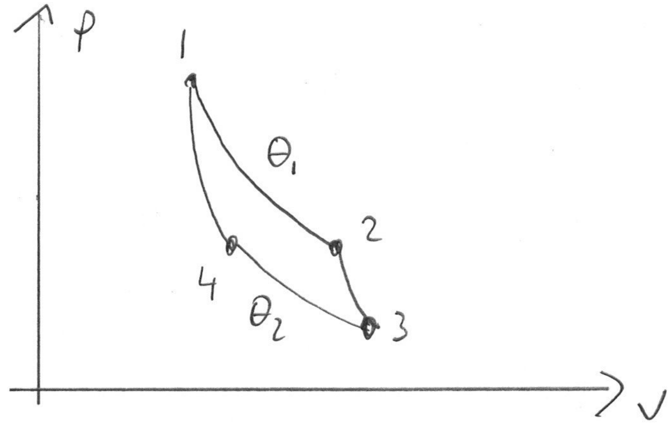
## 2.1




$pV = Nk_B\theta$

$U = \frac{3}{2}Nk_B\theta$

$1\to 2$ isothermal expansion at temperature $\theta_1$

$\theta_1 = \text{const} \Rightarrow U = \text{const} \Rightarrow dU = 0$

$\delta Q = -\delta W = p\,dV$

$Q_1 = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} dV\, \frac{Nk_B\theta_1}{V} = Nk_B\theta_1 \ln\frac{V_2}{V_1}$

$3\to 4$ isothermal compression at temperature $\theta_2$

$Q_2 = \int_{V_3}^{V_4} p\,dV = \int_{V_3}^{V_4} dV\, \frac{Nk_B\theta_2}{V} = Nk_B\theta_2 \ln\frac{V_4}{V_3}$

2-3 adiabatic expansion $\delta Q = 0$

$dU = -p\,dV \qquad \frac{3}{2}Nk_B\,d\theta = -\frac{Nk_B\theta}{V}dV$

$\frac{3}{2}\frac{d\theta}{\theta} = -\frac{dV}{V} \Rightarrow \theta^{3/2}V = \text{Const}$

$\theta_1^{3/2}V_2 = \theta_2^{3/2}V_3$

4-1 adiabatic compression, analogously

$\theta_2^{3/2}V_4 = \theta_1^{3/2}V_1$

$\Rightarrow \frac{V_2}{V_3} = \frac{V_1}{V_4}$ or $\frac{V_1}{V_2} = \frac{V_4}{V_3}$

efficiency

$\eta = 1 + \frac{Q_2}{Q_1}$

$\eta = 1 - \frac{\theta_2}{\theta_1}$

2.2. $dU = TdS + EdP$

$A = U - ST \qquad dA = -SdT + EdP$

$H = U - PE \qquad dH = TdS - PdE$

$G = U - PE - ST \qquad dG = -SdT - PdE$

Maxwell relations

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial E}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial E}{\partial T}\right)_P$$

$$\left(\frac{\partial T}{\partial E}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_E$$

$$\left(\frac{\partial S}{\partial E}\right)_T = \left(\frac{\partial P}{\partial T}\right)_E$$

2.3. $pV = Nk_BT$ , $U = \frac{3}{2}Nk_BT$ , $dU = \delta Q + \delta W$

$$C_V = \left(\frac{\delta Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V - \left(\frac{\delta W}{\partial T}\right)_V = \frac{3}{2}Nk_B$$

$$C_p = \left(\frac{\delta Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p = \frac{3}{2}Nk_B + Nk_B = \frac{5}{2}Nk_B$$

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T = -\frac{1}{V}\frac{\partial}{\partial p}\left(\frac{Nk_BT}{p}\right)_T = \frac{1}{p}$$

for adiabatic processes: $\delta Q = 0 = \frac{3}{2}Nk_B\,dT + \frac{Nk_BT}{V}dV$

$$\frac{3}{2}\frac{dT}{T} = -\frac{dV}{V} \Rightarrow T^{3/2}V = \text{const}$$

$$\Rightarrow pV^{5/3} = \text{const}$$

$$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S = \frac{3}{5}\frac{1}{p}$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{1}{V}\frac{\partial}{\partial T}\left(\frac{Nk_BT}{p}\right)_p = \frac{Nk_B}{pV} = \frac{1}{T}$$