3.1 a) $\langle x \rangle = \int_0^1 dx\, x = \frac{1}{2} = \langle y \rangle$

$$\langle x^2 \rangle = \int_0^1 dx\, x^2 = \frac{1}{3} = \langle y^2 \rangle$$

b) $\sigma_x^2 = \sigma_y^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

c) $\langle z \rangle = \langle x \rangle + \langle y \rangle = 1$

$$\langle z^2 \rangle = \langle (x+y)^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + 2 \langle x \rangle \langle y \rangle$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{2} = \frac{7}{6}$$

$$\sigma_z^2 = \frac{7}{6} - 1 = \frac{1}{6}$$

d) $f_X(k) = f_Y(k) = \int_0^1 dx\, e^{ikx} = \frac{1}{ik} e^{ikx} \Big|_0^1$

$$= \frac{1}{ik}\left(e^{ik} - 1\right)$$

$$P_{\underline{Z}}(z) = \int dx\, dy\, P_{\underline{X}}(x)\, P_{\underline{Y}}(y)\, \delta(z-x-y)$$

$$f_{\underline{Z}}(k) = \int dz\, e^{ikz} \int dx\, dy\, P_{\underline{X}}(x) P_{\underline{Y}}(y)\, \delta(z-x-y)$$

$$= f_{\underline{X}}(k)\, f_{\underline{Y}}(k) = -\frac{1}{k^2}\left(e^{ik} - 1\right)^2$$

Fourier back transformation

$$P_{\underline{Z}}(z) = \begin{cases} 0 & z<0 \text{ or } z<2 \\ z & 0<z<1 \\ 2-z & 1<z<2 \end{cases}$$

<u>Proof:</u> $\int_{-\infty}^{\infty} dz\, e^{ikz} P_{\underline{Z}}(z) = \int_0^1 dz\, z\, e^{ikz} + \int_1^2 dz\, (2-z)\, e^{ikz}$

$$= \frac{1}{k^2}\left(-1 + e^{ik} - ik\, e^{ik}\right) + \frac{1}{k^2}\left(e^{ik} + ik\, e^{ik} - e^{2ik}\right.$$

Diode

$$I = I_0 \left(e^{eV/k_BT} - 1 \right)$$

I between $-I_0$ (for $V \to -\infty$) and
∞ (for $V \to \infty$)

$$P_I(I) = \int dV\, P_V(V)\ \delta\left[I - I_0 \left(e^{eV/k_BT} - 1 \right) \right]$$

$$P_V(V) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2}\frac{V^2}{\sigma^2}}$$

$$\delta[f(V)] = \frac{1}{|f'(V_0)|}\, \delta(V - V_0)$$

only one solution $I(V)$ monotonous

$$e^{eV/k_B} = \frac{I}{I_0} + 1$$

$$\frac{eV}{k_BT} = \ln\left(1 + \frac{I}{I_0}\right) \qquad V_0 = \frac{k_BT}{e} \ln\left(1 + \frac{I}{I_0}\right)$$

$$|f'(V)| = I_0\, e^{eV/k_BT}\, \frac{e}{k_BT} \underset{\text{using } \delta\text{-function}}{=\!=} \frac{e}{k_BT} (I + I_0)$$

$$\boxed{P_I(I) = \frac{1}{\sqrt{2\pi\sigma^2}}\, \frac{k_BT}{e}\, \frac{1}{I + I_0}\, e^{-\frac{1}{2}\left(\frac{k_BT}{e\sigma}\right)^2 \ln^2\left(1 + \frac{I}{I_0}\right)}}$$

$$\langle I \rangle = \int dI \; P_I(I) \; I$$

$$= I_0 \left[e^{\frac{1}{2}\left(\frac{e\sigma}{k_B T}\right)^2} - 1 \right]$$

most probable I

$$\frac{\partial}{\partial I} P(I) = 0 \qquad \text{use variable} \quad x = \frac{I + I_0}{I_0}$$

$$\frac{\partial}{\partial x} \bar{P}(x) = 0$$

$$-\frac{1}{x^2} e^{-\frac{1}{2} A^2 \ln^2(x)} + \frac{1}{x} e^{-\frac{1}{2} A^2 \ln^2(x)} \left(-\frac{1}{2} A^2 \, 2 \ln(x)\right) \frac{1}{x} = 0$$

$$1 = -A^2 \ln(x) \qquad x = e^{-\frac{1}{A^2}}$$

$$I_p = I_0 (x_p - 1) = I_0 \left(e^{-\left(\frac{e\sigma}{k_B T}\right)^2} - 1 \right)$$

3.3 a) $P(5) = \frac{1}{2^{10}} \binom{10}{5} = \frac{252}{1024} = 0.2461$

$P(4) = \frac{1}{2^{10}} \binom{10}{4} = \frac{210}{1024} = 0.2051$

$P(4)/P(5) = 5/6 = 0.8333$

3b) $P(500) = \frac{1}{2^{1000}} \binom{1000}{500} = \frac{1}{2^{1000}} \frac{1000!}{500!\,500!}$

$$\begin{aligned}\ln P(500) &= -1000 \ln(2) + 1000 \ln(1000) - \cancel{1000} \\ &\quad - 500 \ln(500) + \cancel{500} \quad - 500 \ln(500) + \cancel{500} \\ &= -1000 \left[-\ln(2) + \ln(1000) - \ln(500) \right] \\ &= 0\end{aligned}$$

$P(500) \approx 1$ (<u>not</u> a very good approx.)

$$\begin{aligned}\ln P(400) &= -1000 \ln(2) + 1000 \ln(1000) \\ &\quad - 400 \ln(400) - 600 \ln(600) \\ &= -20.13\end{aligned}$$

$P(400) \approx 1.81 \times 10^{-9}$

$P(400)/P(500) \approx 1.81 \times 10^{-9}$