Quantum phase transitions and disorder: Griffiths singularities, infinite randomness, and smearing

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- Phase transitions and quantum phase transitions
- Effects of impurities and defects: the common lore
- Rare regions, Griffiths singularities and smearing
- Experiments in disordered superconductors and itinerant magnets
- Classification of weakly disordered phase transitions

Acknowledgements

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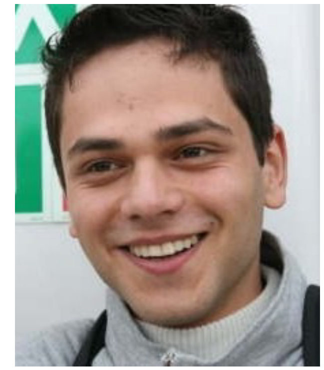

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Collaborators: Experiment

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(Kent State)

Istvan Kezsmarki
(TU Budapest)

Theory

Jose Hoyos
(Sao Paulo)

Phase transitions: the basics

Phase transition:

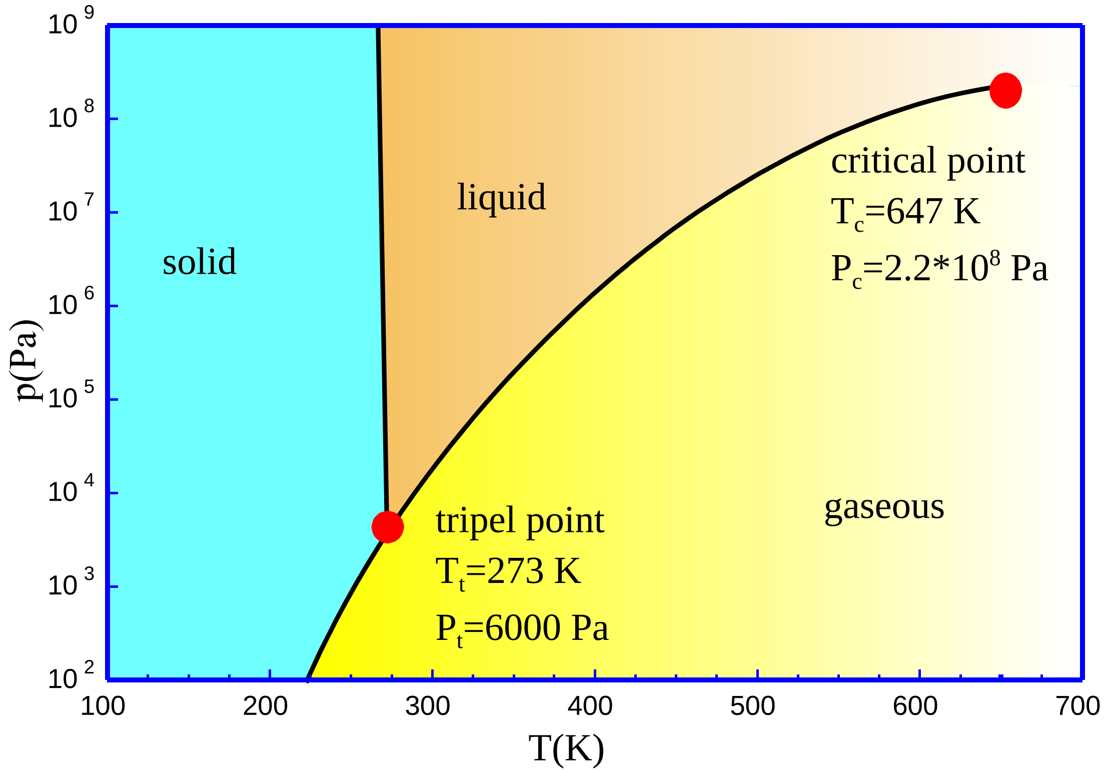
- singularity in free energy
- occurs in macroscopic systems

1st order transition:

- phase coexistence, latent heat
- finite correlation length and time

Continuous transition:

- no phase coexistence, latent heat
- diverging correlations

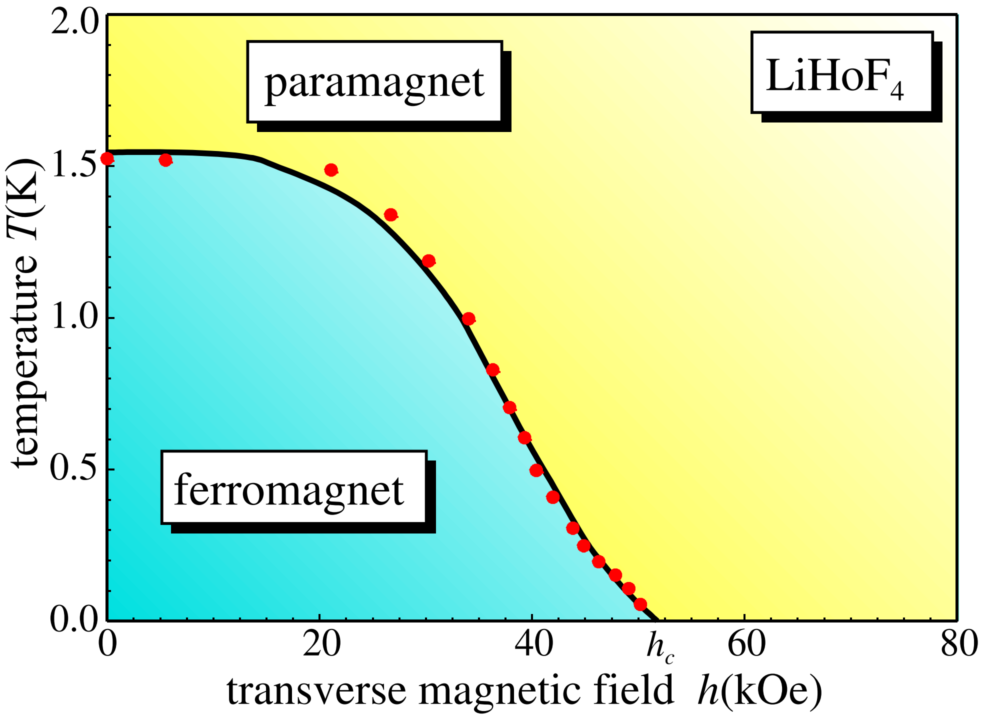


Critical behavior:

- diverging **correlation length** $\xi \sim |T - T_c|^{-\nu}$ **and time** $\xi_\tau \sim \xi^z \sim |T - T_c|^{-\nu z}$
- power-laws in thermodynamic observables: $\Delta\rho \sim |T - T_c|^{\beta}, \ \kappa \sim |T - T_c|^{-\gamma}$
- critical exponents are **universal** $=$ independent of microscopic details

Quantum phase transitions

- occur at **zero temperature** as function of pressure, magnetic field, ...
- driven by **quantum** rather than thermal fluctuations



Transverse-field Ising model

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

transverse magnetic field induces spin flips via $\sigma^x = \sigma^+ + \sigma^-$

transverse field suppresses magnetic order

Quantum to classical mapping:
- maps QPT in d dimensions to classical PT in $d+1$ dimensions
- imaginary time plays role of additional dimension

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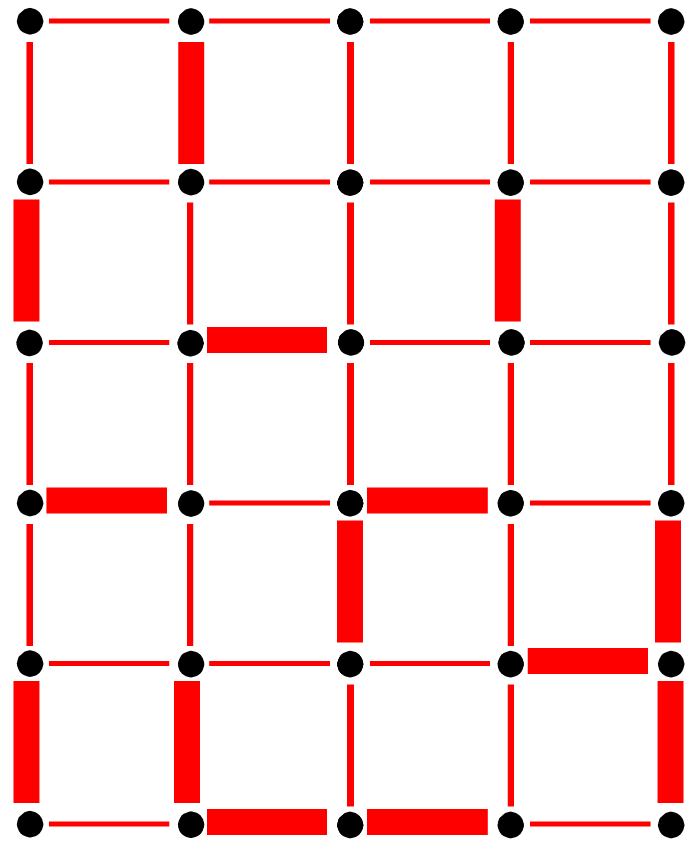
Phase transitions and disorder

- system undergoing **classical**, **quantum**, or **nonequilibrium** phase transition
- real systems always contain **impurities, defects** and other types of **disorder**



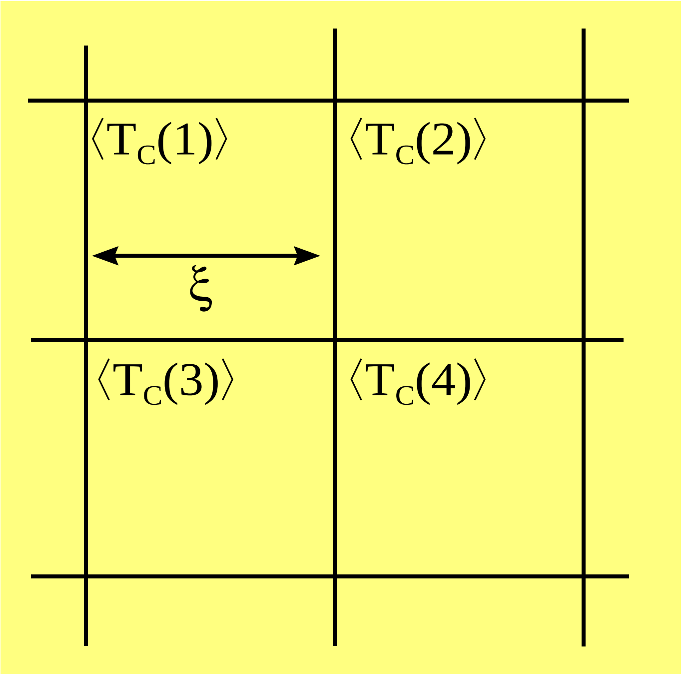
Weak (random-T_c, random-mass) disorder:

- **spatial variation** of coupling strength
- locally favors one phase over the other
- does **not** break order-parameter symmetries
- **no** change in character of the bulk phases

Will the phase transition remain sharp or become smeared?
Will the order of the transition change
Will the critical behavior change?

Harris criterion

Harris: stability of clean critical point
variation of average local $T_c(i)$ between correlation volumes must be smaller than distance from global T_c



variation of average $T_c(i)$ in volume ξ^d
$\Delta T_c(i) \sim \xi^{-d/2}$

global distance from critical point
$T - T_c \sim \xi^{-1/\nu}$

$\Delta T_c(i)/(T - T_c) \to 0$ at criticality

$$d\nu > 2$$

- if clean critical point fulfills Harris criterion $\Rightarrow$ stable against disorder
- system is **asymptotically clean** as inhomogeneities vanish at large length scales
- macroscopic observables are **self-averaging**
- example: **3D classical Heisenberg magnet**: $\nu = 0.711$

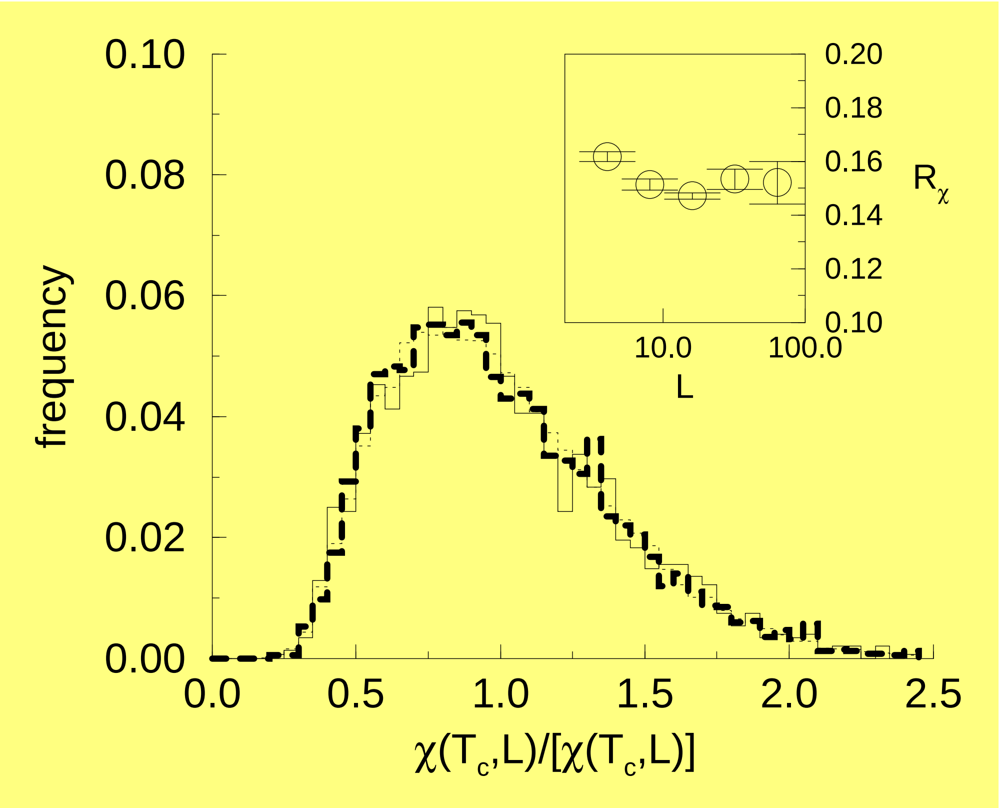
extension to general spatio-temporal disorder: T.V and R. Dickman, PRE **93**, 032143 (2016)

Finite-disorder critical points

if critical point violates Harris criterion $\Rightarrow$ unstable against disorder

Common lore:

- new, different critical point which fulfills $d\nu > 2$
- inhomogeneities finite at all length scales (**"finite disorder"**)
- macroscopic observables **not self-averaging**
- example: **3D classical Ising magnet**: clean $\nu = 0.627 \quad \Rightarrow \quad$ dirty $\nu = 0.684$



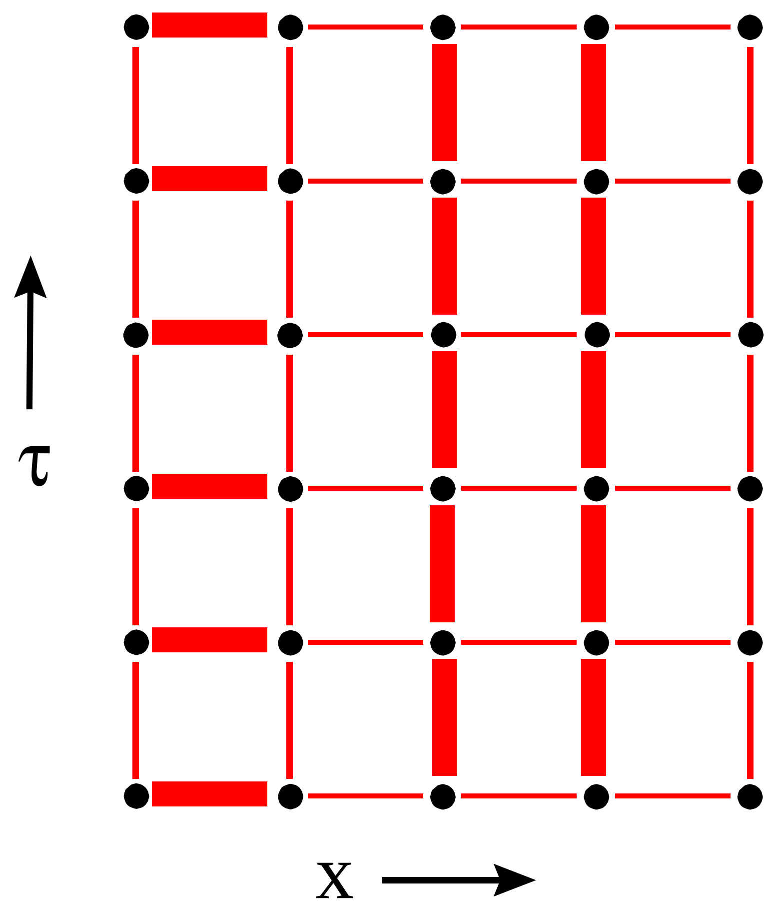
Distribution of critical susceptibilities of 3D dilute Ising model
(Wiseman + Domany 98)

Disorder and quantum phase transitions

Disorder is quenched:

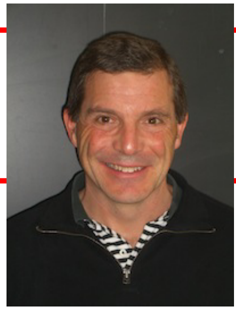
- impurities are time-independent
- disorder is **perfectly correlated** in imaginary time direction

⇒ correlations **increase** the effects of disorder ("it is harder to average out fluctuations")



Disorder generically has stronger effects on quantum phase transitions than on classical transitions

Random transverse-field Ising model

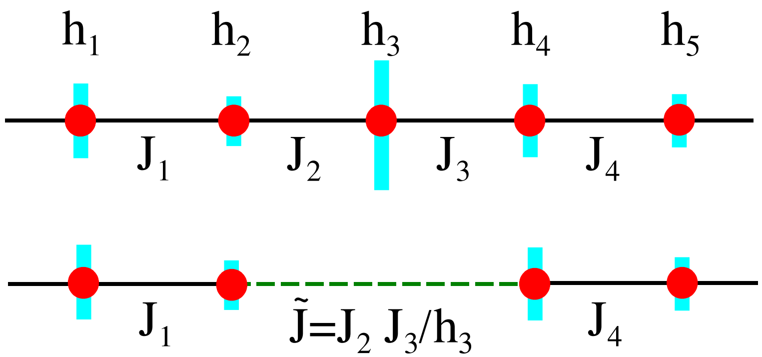


$$H = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^x$$

nearest neighbor interactions J_{ij} and transverse fields h_i both random

Strong-disorder renormalization group:

- Ma, Dasgupta, Hu (1979), Fisher (1992, 1995)
- in each step, integrate out **largest** of all J_{ij}, h_i
- cluster aggregation/annihilation process
- exact in the limit of large disorder



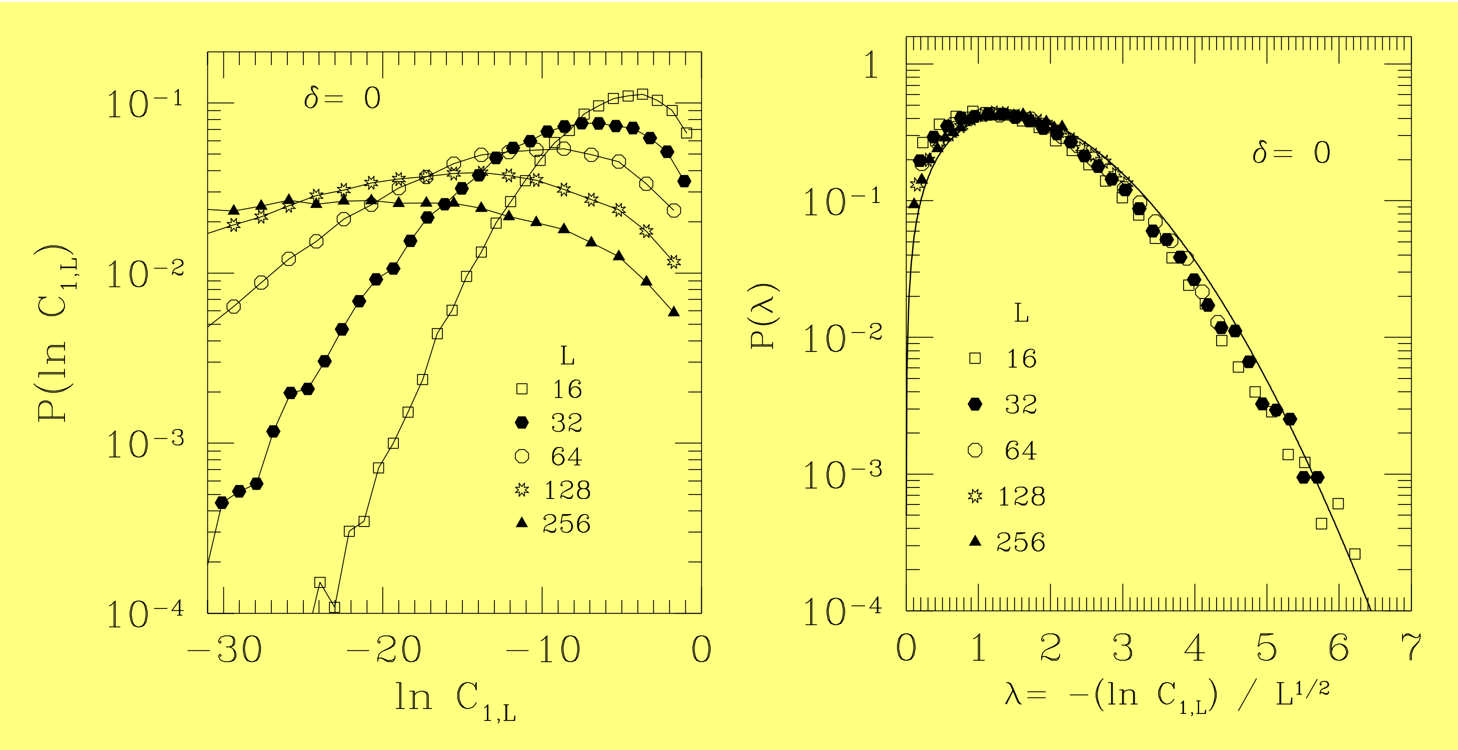
Analytical solution in 1+1 dimensions:

- flow equations for entire probability distributions $P(J), R(h)$
- under renormalization, relative width of the distributions **diverges**

  $\Rightarrow$ disorder **increases without limit**

Infinite-disorder critical point

- distributions of macroscopic observables become infinitely broad
- **average** and **typical** values drastically different
  correlations: $G_{av} \sim r^{-\eta}$, $\quad -\log G_{typ} \sim r^{\psi}$
- averages dominated by **rare events**
- **extremely slow** dynamics $\log \xi_\tau \sim \xi^\mu$ (**activated dynamical scaling**)



Probability distribution of end-to-end correlations in a random quantum Ising chain
(Fisher + Young 98)

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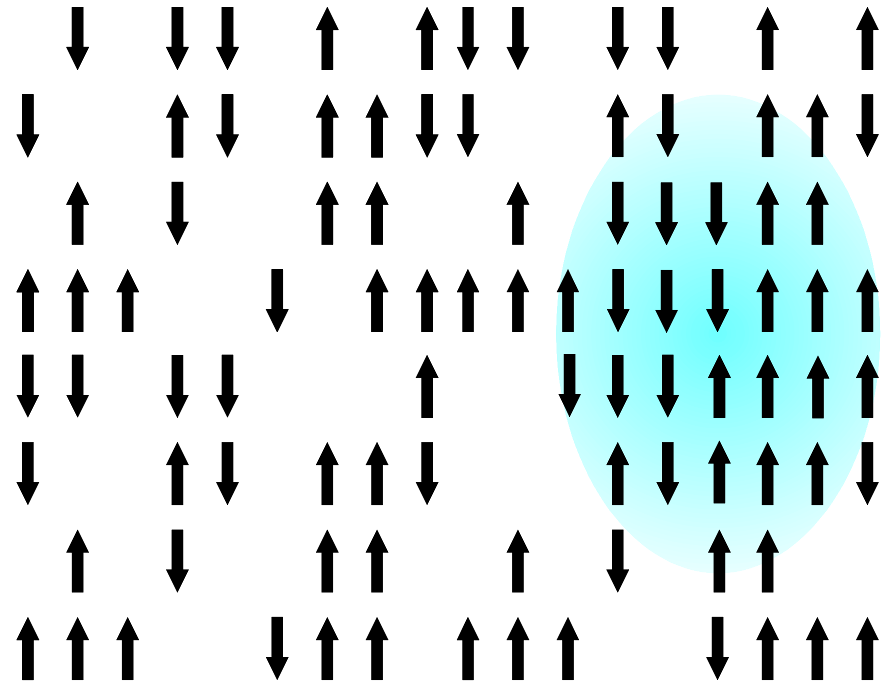
Rare regions in a classical dilute ferromagnet

- critical temperature T_c is **reduced** compared to clean value T_{c0}
- for $T_c < T < T_{c0}$:
  no global order but **local order** on **rare regions** devoid of impurities
- rare region probability **exponentially small**
  $p(L) \sim e^{-cL^d}$

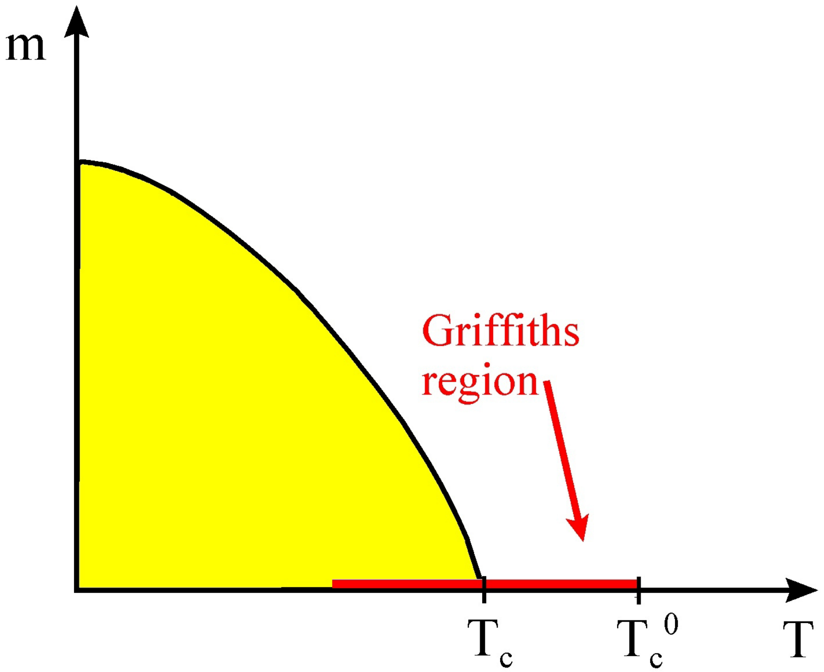


Thermodynamics of rare regions

- rare regions cannot order statically but act as large **superspins**
- **very slow** dynamics, **enhanced** thermodynamic response

Can rare regions dominate thermodynamics of the entire system?

Griffiths region or Griffiths "phase"



Griffiths:

rare regions lead to singular free energy everywhere in the interval $T_c < T < T_{c0}$

Rare region susceptibility:

- susceptibility of single RR: $\chi \lesssim L^{2d}/T$
- sum over all RRs:

$$\chi_{RR} \sim \int dL\, e^{-cL^d} L^{2d}$$

- essential singularity
- large regions make negligible contribution

In generic classical systems:

Thermodynamic Griffiths effects are weak and essentially unobservable

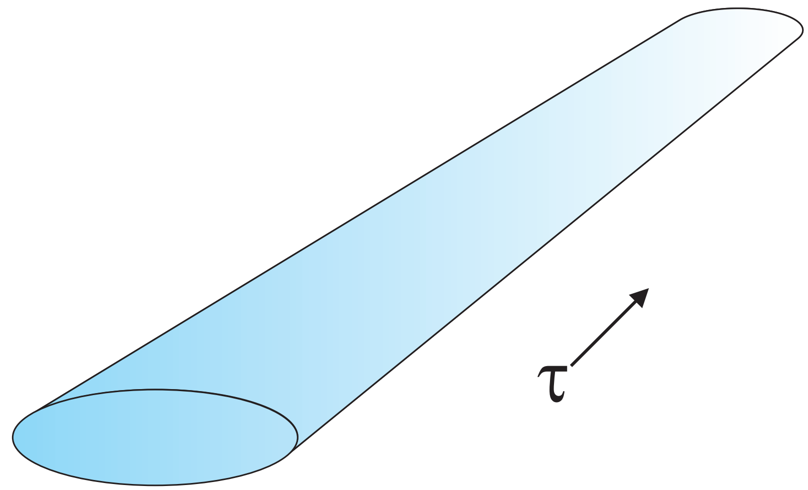
Long-time dynamics can be dominated by rare regions

Quantum Griffiths effects

Quantum phase transitions:

- rare regions are finite in space but **infinite in imaginary time**
- fluctuations **even slower** than in classical case

Griffiths singularities enhanced

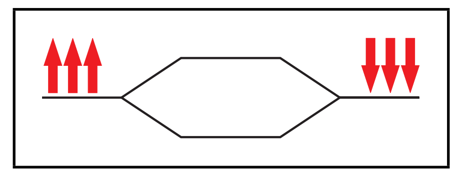


rare region at a quantum phase transition

Transverse-field Ising systems:

- susceptibility of rare region: $\chi_{loc} \sim \Delta^{-1} \sim e^{aL^d}$

  $\chi_{RR} \sim \int dL\, e^{-cL^d} e^{aL^d}$ can **diverge** inside Griffiths region



- power-law **quantum Griffiths singularities**

  susceptibility: $\chi_{RR} \sim T^{d/z'-1}$, specific heat: $C_{RR} \sim T^{d/z'}$

z' is **continuously varying** Griffiths dynamical exponent, **diverges** at criticality

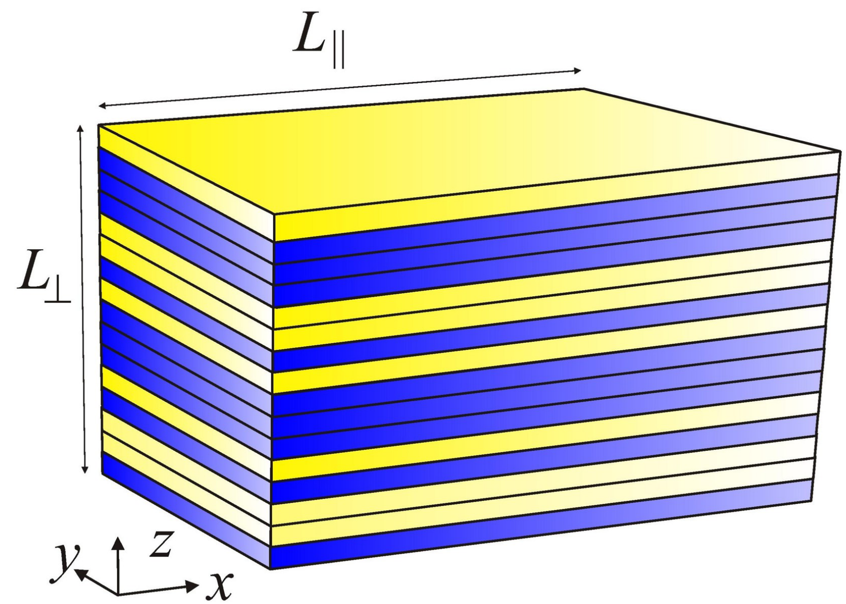
Connection between Harris criterion and Griffiths sing., T.V. + J.A. Hoyos, PRL **112**, 075702 (2014)

Smeared phase transitions

Randomly layered classical magnet:

- random layers of two different ferromagnets
- rare regions are **thick 2d slabs** of the material with higher T_c
- 2d (Ising) magnets have **true phase transition**

⇒ global magnetization develops **gradually** as rare regions order **independently**



global phase transition is smeared by disorder

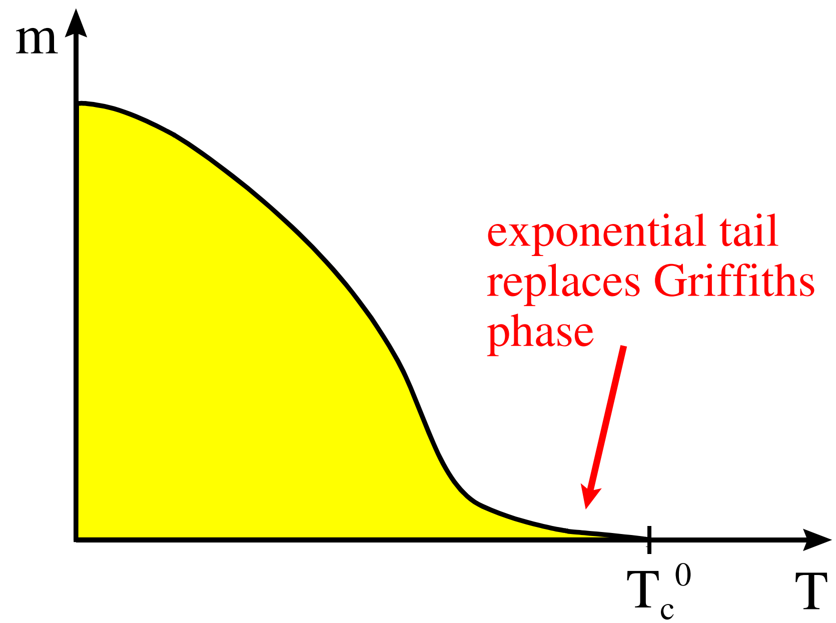
Smeared quantum phase transitions:

- if isolated rare region develops **static order parameter** ⇒ transition **smeared**
- **example:** itinerant Ising magnet (order parameter **damped** by coupling to electrons, this prevents rare regions from tunneling)

T.V., PRL **90**, 107202 (2003), J.A. Hoyos and T.V., PRL **100**, 240601 (2008)

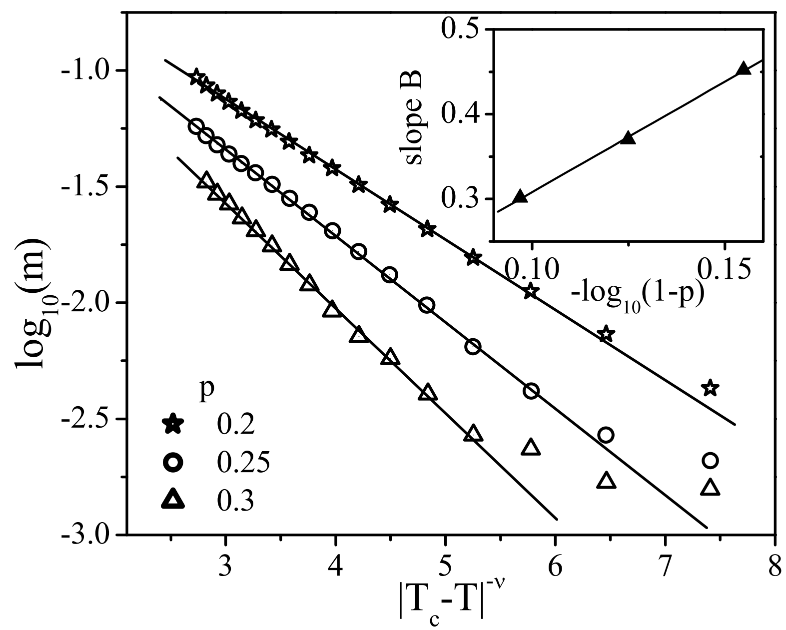
Magnetization tail of smeared transition

- tail produced by largest rare regions (thickest slabs)
- slab transition temperature $T_c(L) < T_c^0$ (T_c^0 = higher of the two bulk T_c)
- **finite size scaling:** $|T_c(L) - T_c^0| \sim L^{-\phi}$ (ϕ = clean shift exponent)
- **probability** for slab devoid of weak planes: $w \sim e^{-cL}$



Magnetization tail for $T \to T_c^0 -$

$$m(T) \sim \exp(-B\ |T - T_c^0|^{-1/\phi})$$



R. Sknepnek and T.V., PRB **69**, 174410 (2004)

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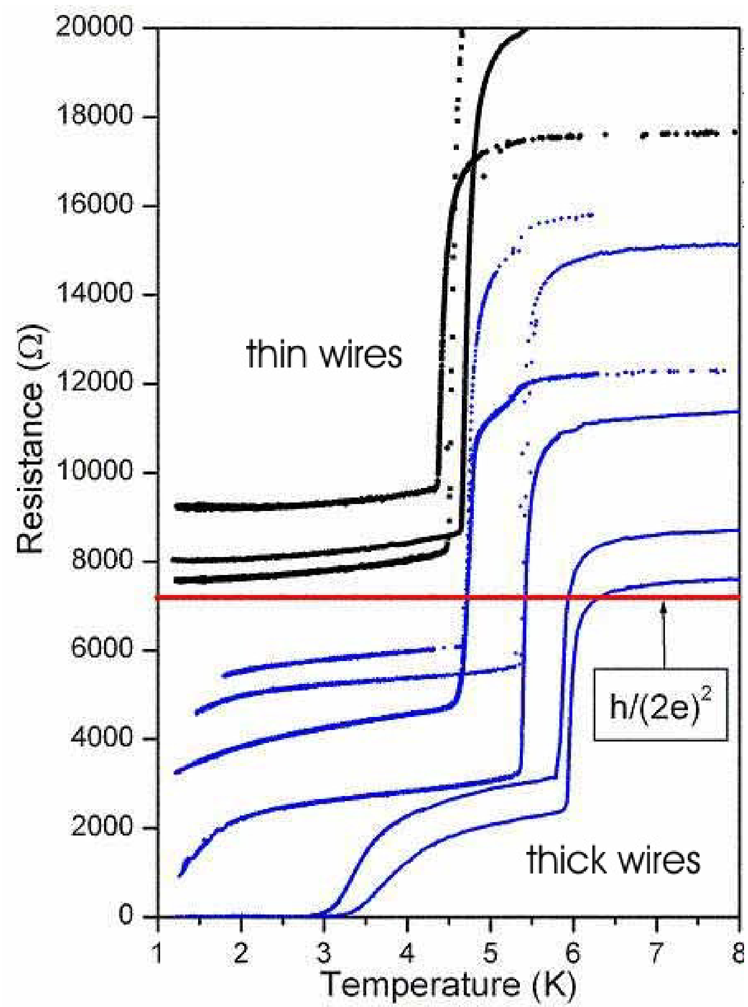
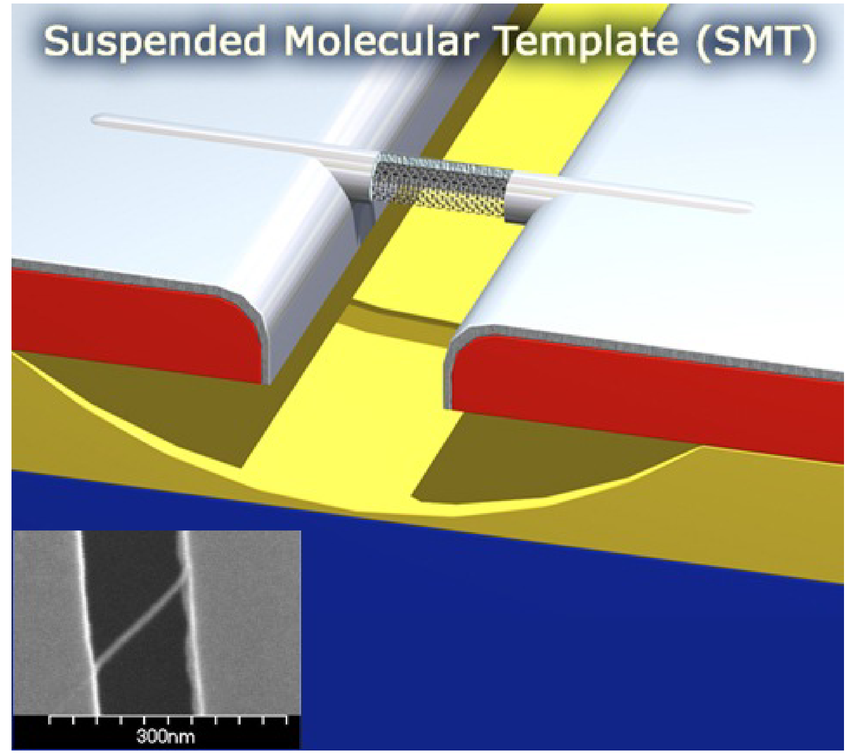
Superconductor-metal quantum phase transition in nanowires

- ultrathin MoGe wires (width $\sim$ 10 nm)
- molecular templating using a single carbon nanotube [Bezryadin group, UIUC]





- pair breaking by surface magnetic impurities
- creates **disorder** and **dissipation**

superconductor-metal QPT as function of wire thickness

Quantum Landau-Ginzburg-Wilson theory

- number of transport channels (states transverse to wire) is large, $N_\perp \gg 1$
  $\Rightarrow$ motion of (unpaired) electrons is **three-dimensional**
- wire width $\approx$ 10nm $\sim$ coherence length,   wire length $\approx$ 500nm
  $\Rightarrow$ superconducting critical fluctuations are **one-dimensional**

Free energy functional:

$$S = T \sum_{\mathbf{q},\omega_n} \left(r + \xi_0^2 \mathbf{q}^2 + \gamma\, |\omega_n| \right) |\varphi(\mathbf{q},\omega_n)|^2 + \frac{u}{2N} \int \mathrm{d}^d x \mathrm{d}\tau\ \varphi^4(\mathbf{x},\tau)$$

To apply strong-disorder RG, discretize space:

$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|)\, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i\, \phi_i(-\omega_n)\, \phi_{i+1}(\omega_n)$$

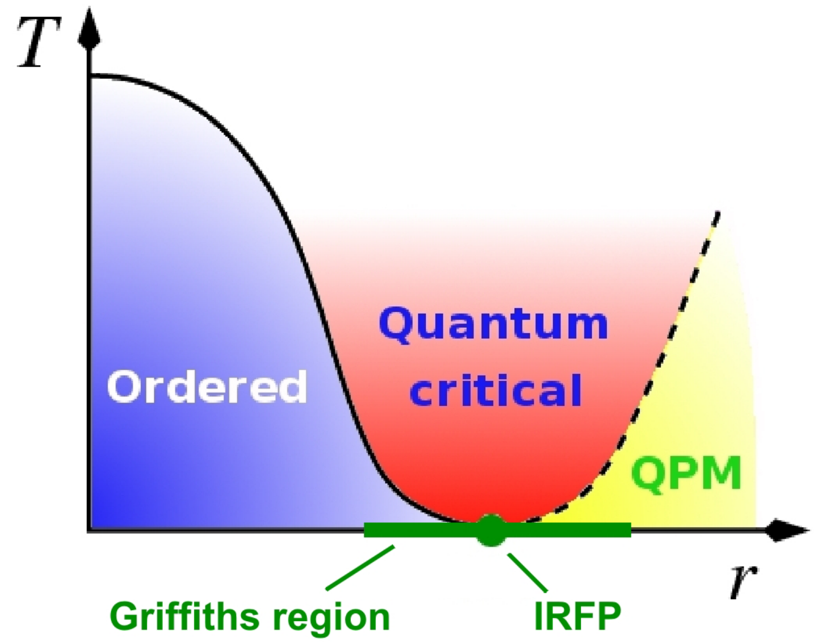
$\Rightarrow$ chain of coupled superconducting grains, coupled by Josephson interactions

$\Rightarrow$ disorder: ϵ_i, γ_i, J_i random variables

Strong-disorder renormalization group

Competing local energies:

- local **"energy gaps"** ϵ_i, favoring normal phase
- **bonds** J_i (Josephson couplings), favoring superconducting phase



$$T_c \sim \exp(-\text{const}\,|r|^{-\nu\psi})$$

Infinite-randomness critical point:

- distributions $P(J)$ and $R(\epsilon)$ become **infinitely broad**
- universality class of random transverse-field Ising model
- critical exponents known **exactly** in 1D
- **activated** dynamical scaling, $\log \xi_t \sim \xi^\psi$ ($\psi = 1/2$ in 1D)
- **higher dimensions:** same activated scaling scenario, exponents known numerically

J.A. Hoyos, C. Kotabage and T.V., PRL **99**, 230601 (2007), PRB **79**, 024401 (2009)

Critical behavior and Griffiths singularities

Specific heat:

$$C(r,T) = \left(\ln \frac{T_0}{T}\right)^{-d/\psi} \Phi_C \left(r^{\nu\psi} \ln \frac{T_0}{T}\right)$$

Griffiths phase:

$$C(r,T) \sim T^{d/z'}$$

Griffiths dynamical exponent $z' \sim r^{-\nu\psi}$ **diverges** at criticality

Ordered Griffiths phase:

long-range order but vanishing stiffness **anomalous elasticity**:

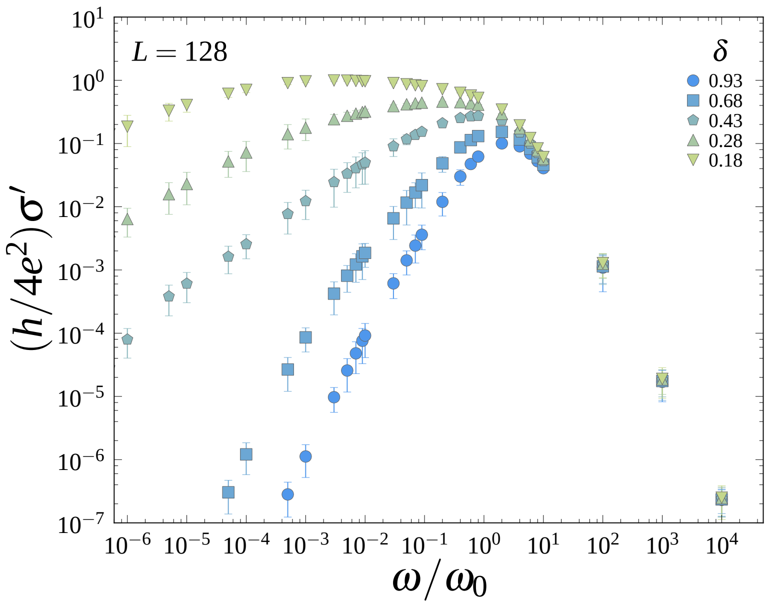
$$f(\Theta) - f(0) \sim \Theta^2 L^{-(1+z)} \qquad (z > 1)$$

P. Mohan, P.M. Goldbart, R. Narayanan, J. Toner and T.V, PRL **105**, 085301 (2010)

Dynamical (optical) conductivity:

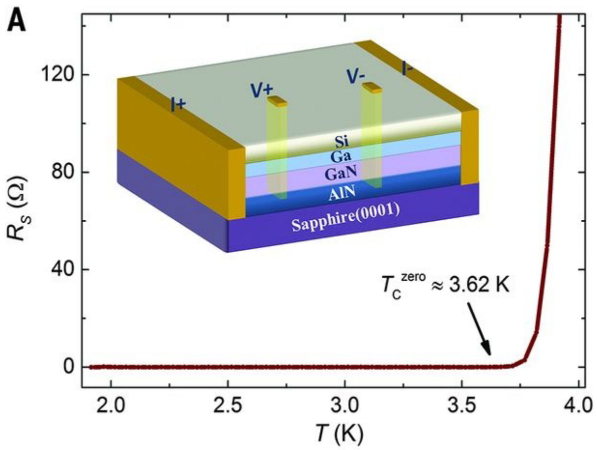
- calculated from **Kubo formula**
- include **vector potential** in SDRG

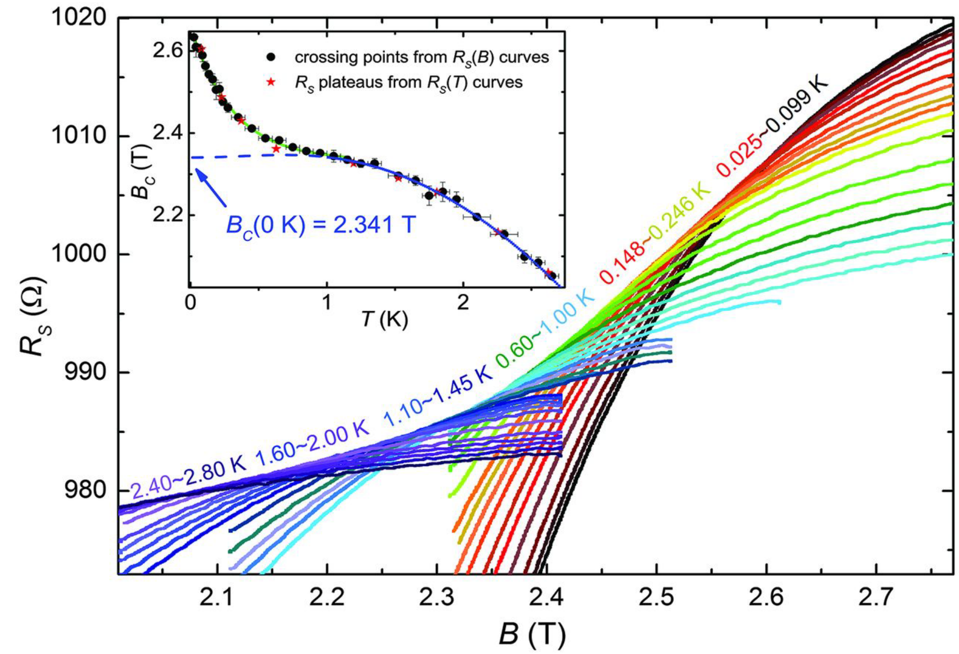
$$\sigma'(r,\omega) = \frac{4e^2}{h}\left(\ln \frac{\omega_0}{\omega}\right)^{1/\psi} \Phi_\sigma \left(r^{\nu\psi} \ln \frac{\omega_0}{\omega}\right)$$



A. Del Maestro, B. Rosenow, J.A. Hoyos and T.V., PRL **105**, 145702 (2010)

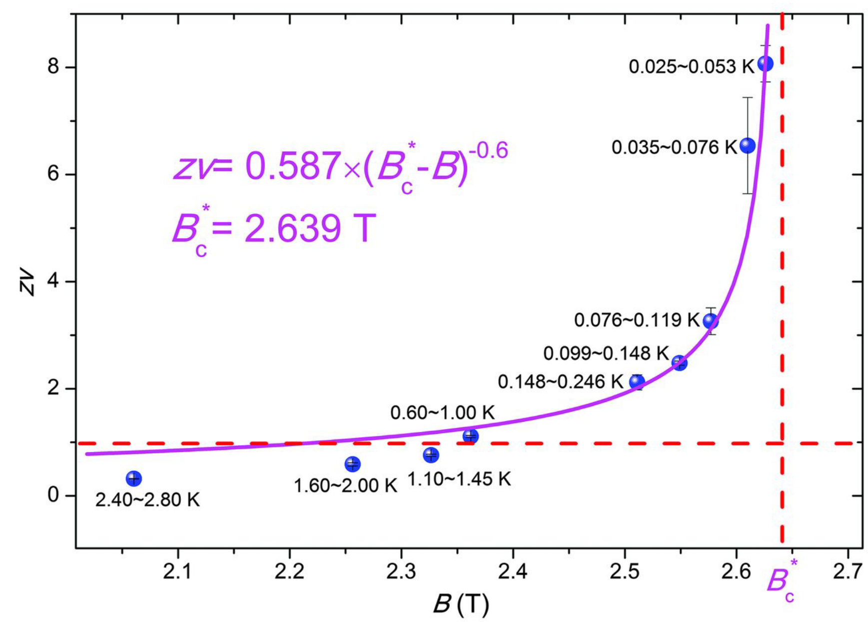
Experiment: ultrathin Ga films





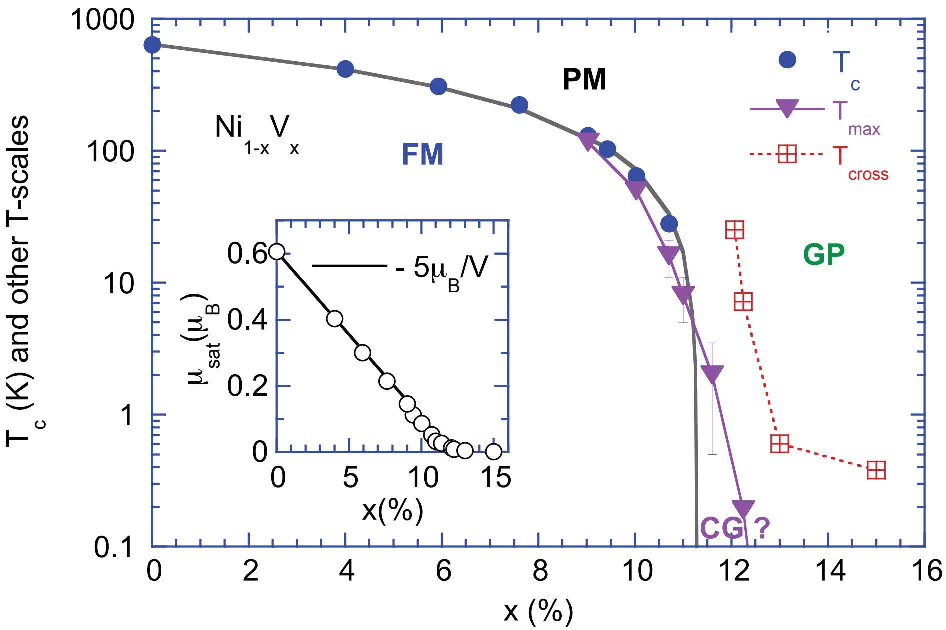
Xing et al., Science 350, 542 (2015)

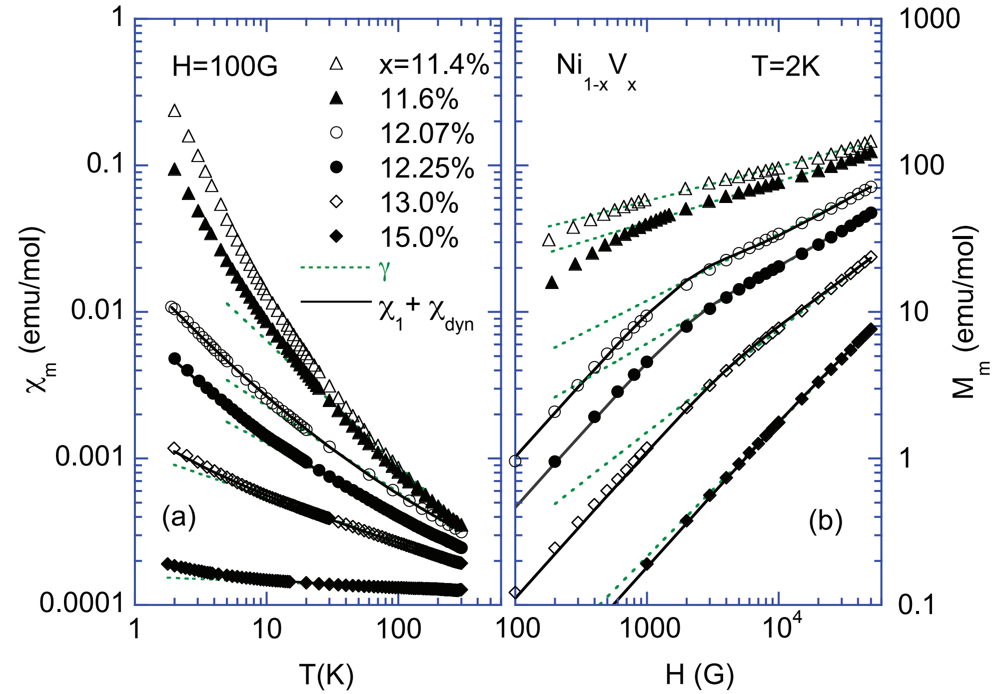
- superconductivity below $T_c \approx 3.62$K, suppressed by magnetic field
- field-driven QPT well described by **2D infinite-randomness critical point**
- dynamical exponent **diverges** as $z \sim |B - B_c|^{-\nu\psi}$ with $\nu \approx 1.2, \psi \approx 0.5$

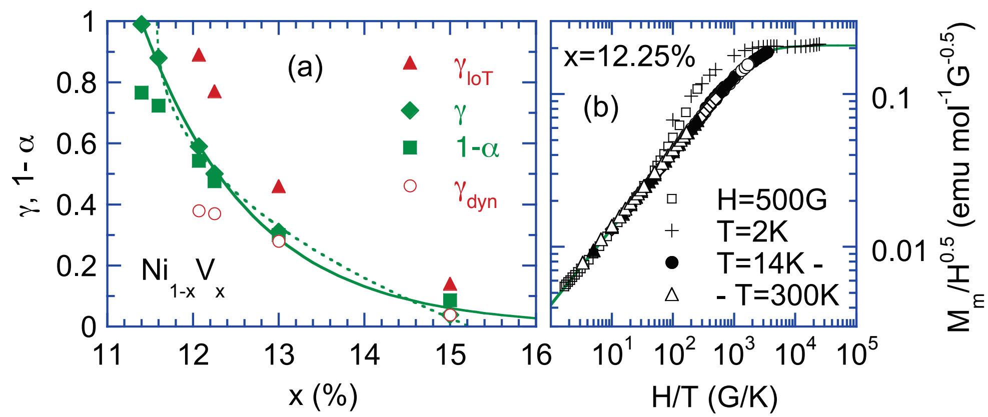


Exponent values from MC simulations by T.V., A. Farquhar, J. Mast, PRE **79**, 011111 (2009)

Ferromagnetic Griffiths singularities in $Ni_{1-x}V_x$

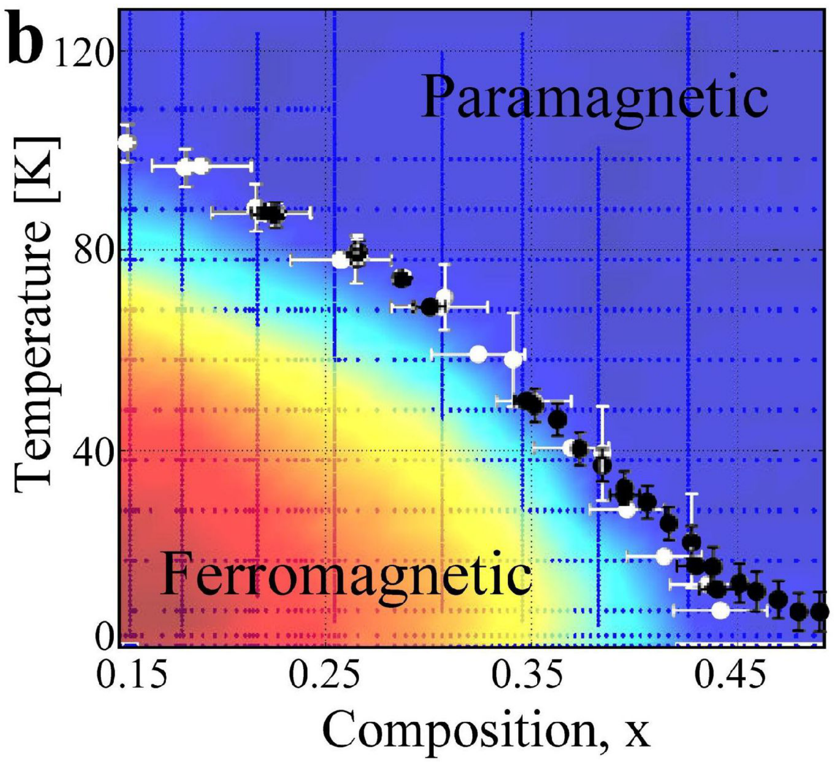


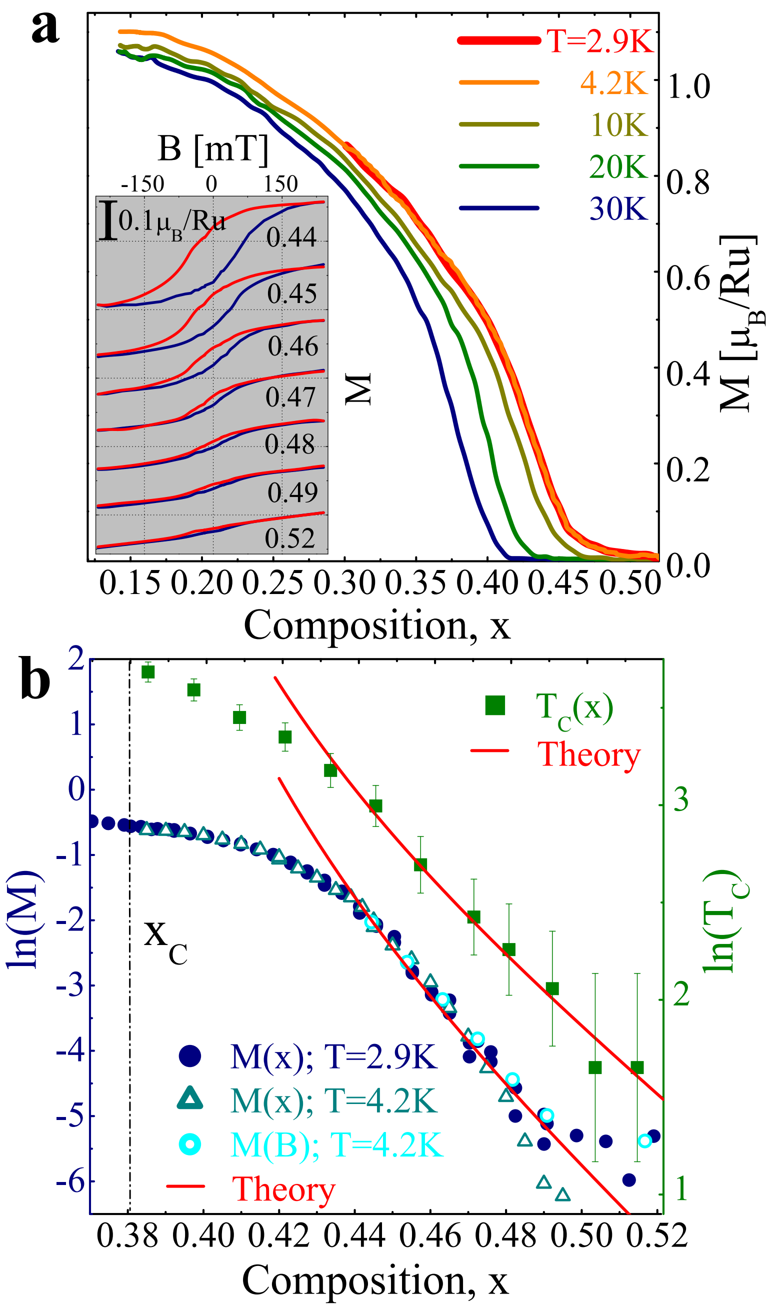




S. Ubaid-Kassis, T. V., A. Schroeder, PRL **104**, 066402 (2010)
A. Schroeder, S. Ubaid-Kassis, T.V., JPCM **23**, 094205 (2011)
D. Nozadze + T.V., PRB **85**, 174202 (2012)
R. Wang et al., PRL **118**, 267202 (2017)

Smeared quantum phase transition in $Sr_{1-x}Ca_xRuO_3$





Magnetization and T_c in tail:

$$M, T_c \sim \exp\left[-C\frac{(x-x_c^0)^{2-d/\phi}}{x(1-x)}\right]$$

L. Demkó et al, PRL **108**, 185701 (2012)
F. Hrahsheh et al., PRB **83**, 224402 (2011)
C. Svoboda et al., EPL **97**, 20007 (2012)

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Disorder at phase transitions: two frameworks

- fate of average disorder strength under coarse graining
- importance of rare regions and strength of Griffiths singularities

Recently:

- general relation between **Harris criterion and rare region physics**
  T.V. + J.A. Hoyos, Phys. Rev. Lett. **112**, 075702 (2014), Phys. Rev. E **90**, 012139 (2014)
- **below** d_c^+, same inequality, $d\nu > 2$, governs relevance or irrelevance of disorder and fate of the Griffiths singularities

Class	RR dimension	Subclass	Harris criterion	Griffiths effects	Critical behavior of disordered system
A	$d_{\mathrm{RR}} < d_c^-$	A1	$d\nu > 2$	weak exponential	clean
		A2	$d\nu < 2$	weak exponential	conventional finite disorder
B	$d_{\mathrm{RR}} = d_c^-$	B1	$d\nu > 2$	power law, z' remains finite	clean
		B2	$d\nu < 2$	power law, z' diverges	strong or infinite randomness
C	$d_{\mathrm{RR}} > d_c^-$			rare regions freeze	smeared transition

- **above** d_c^+, behavior is even richer
- relevance of rare regions depends on inequality $d_c^+\nu > 2$

Conclusions

- even weak disorder can have surprisingly strong effects on a phase transition
- **rare regions** play a much bigger role at quantum phase transitions than at classical transitions
- classification of Griffiths phenomena according to **effective dimensionality** of rare regions
- experimental evidence for **quantum Griffiths singularities** and **smeared phase transitions** has been found at magnetic and superconducting quantum phase transitions in disordered metals

Quenched disorder at quantum phase transitions leads to a rich variety of new effects and exotic phenomena

Reviews: T.V., J. Phys. A **39**, R143 (2006); J. Low Temp. Phys. **161**, 299 (2010); AIP Conf. Proc. **1550**, 188 (2013); Ann. Rev. Cond. Mat. Phys., to appear 2018

Imaginary time and quantum to classical mapping

Classical partition function: statics and dynamics decouple

$Z = \int dpdq\ e^{-\beta H(p,q)} = \int dp\ e^{-\beta T(p)} \int dq\ e^{-\beta U(q)} \sim \int dq\ e^{-\beta U(q)}$

Quantum partition function: statics and dynamics coupled

$Z = \mathrm{Tr} e^{-\beta \hat{H}} = \lim_{N\to\infty} (e^{-\beta \hat{T}/N} e^{-\beta \hat{U}/N})^N = \int D[q(\tau)]\ e^{S[q(\tau)]}$

imaginary time τ acts as additional dimension
at $T = 0$, the extension in this direction becomes infinite

Caveats:

- mapping holds for thermodynamics only
- resulting classical system can be unusual and anisotropic ($z \neq 1$)
- if quantum action is not real, extra complications may arise, e.g., Berry phases

Strong-disorder renormalization group

- introduced by Ma, Dasgupta, Hu (1979), further developed by Fisher (1992, 1995)
- **asymptotically exact** if disorder distribution becomes broad under RG

Basic idea: Successively integrate out the local high-energy modes and renormalize the remaining degrees of freedom.

Discretized large-N action: (ϵ_i, γ_i, J_i: random variables)

$$S = T \sum_{i,\omega_n} (\epsilon_i + \gamma_i |\omega_n|) \, |\phi_i(\omega_n)|^2 - T \sum_{i,\omega_n} J_i \, \phi_i(-\omega_n) \, \phi_{i+1}(\omega_n)$$

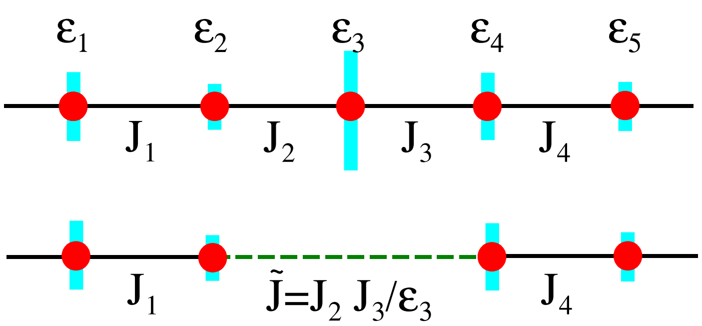
the competing local energies are:

- bonds (Josephson couplings) J_i, favoring ordered phase
- local "energy gaps" ϵ_i, favoring disordered phase

$\Rightarrow$ in each RG step, integrate out largest among all J_i and ϵ_i

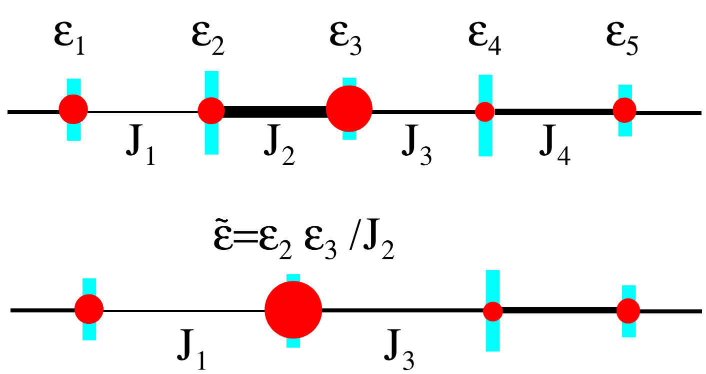
J.A. Hoyos, C. Kotabage and T.V., PRL **99**, 230601 (2007), PRB **79**, 024401 (2009)

RG recursions and flow equations



if largest energy is a gap, e.g., $\epsilon_3 \gg J_2, J_3$:

- site 3 is removed from the system
- bonds treated in 2nd order perturbation theory
- **renormalized bond** $\tilde{J} = J_2 J_3 / \epsilon_3$



if largest energy is a bond, e.g., $J_2 \gg \epsilon_2, \epsilon_3$:

- rotors of sites 2 and 3 are parallel
- replaced by single rotor, moment $\tilde{\mu} = \mu_2 + \mu_3$
- **renormalized gap** $\tilde{\epsilon} = \epsilon_2 \epsilon_3 / J_2$

flow equations for probability distributions $P(J)$ and $R(\epsilon)$

$$-\frac{\partial P}{\partial \Omega} = [P(\Omega) - R(\Omega)]\, P + R(\Omega) \int dJ_1 dJ_2\, P(J_1) P(J_2)\, \delta\left(J - \frac{J_1 J_2}{\Omega}\right)$$

$$-\frac{\partial R}{\partial \Omega} = [R(\Omega) - P(\Omega)]\, R + P(\Omega) \int d\epsilon_1 d\epsilon_2\, R(\epsilon_1) R(\epsilon_2)\, \delta\left(\epsilon - \frac{\epsilon_1 \epsilon_2}{\Omega}\right)$$

Fixed points

If bare distributions do not overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: no clusters formed – disordered phase

$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: all sites connected – ordered phase

If bare distributions do overlap:

$\langle \ln \epsilon \rangle > \langle \ln J \rangle$: rare clusters – disordered Griffiths phase

$\langle \ln \epsilon \rangle < \langle \ln J \rangle$: rare "holes" – ordered Griffiths phase

$\langle \ln \epsilon \rangle = \langle \ln J \rangle$: cluster aggregation and decimation balance at all energies – **critical point**

$$\mathcal{P}(\zeta) = \frac{1}{\Gamma} e^{-\zeta/\Gamma}, \quad \mathcal{R}(\beta) = \frac{1}{\Gamma} e^{-\beta/\Gamma}$$

log. variables $\zeta = \ln(\Omega/J)$, $\beta = \ln(\Omega/\epsilon)$, $\Gamma = \ln(\Omega_0/\Omega)$

Distributions become infinitely broad

$\Rightarrow$ infinite-randomness critical point